Critical Variables under Stochastic Disturbances

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***Abstract*—In the paper discusses the properties of the critical variables, for which the probability to fall within the specified interval is less than the required one. To increase the probability that the variable lies in the feasible region a control vector with the minimum number of components is chosen from a set of possible controls, using the method of contribution factor The presented numerical results on the example of test and real networks demonstrate the efficiency of the proposed approaches to the electric power system operation control.**

Keywords—Critical variables, probabilistic load flow, contribution factors, controls.

I. Introduction

A large response of variables to a disturbance is significant in the case if it changes some criterion of power system operation, for example the criterion of operation feasibility.

To increase the probability that the variable lies in the feasible region it is necessary either to reinforce the network, which will improve the Jacobian matrix conditioning, reduce the response of the variables to the disturbance and expand the feasible region, or to find the appropriate controls to make the variable mean value shift inside the feasible region.

The required probability is provided by iteratively and consecutively solving the problem of probabilistic load flow [1] and the problem of determining feasible operating conditions by the deterministic equivalent method, which implies searching for a feasible solution with a shift of the critical variable mean value inside the feasible region.

The critical variables are the variables for which the probability to fall within the specified interval is less than the required one. If the solution exists, a control vector with the minimum number of components is chosen from a set of possible controls, using the method of contribution factor [2], and the required increment in the control vector is determined, which increases the probability that the critical variable lies in the feasible region.

The study to be mentioned among the first to consider constraints in the calculation of probabilistic load flows is [3]. The need to solve the indicated problem by the procedure for calculating constrained optimal power flow has resulted in the development of the methods that combine deterministic and probabilistic approaches. The theoretical foundations of this approach called the method of deterministic equivalent are presented in [4].

II. Stochactic Control Problem

A strong response of the sensor variable [5], [6] to the disturbance is not dangerous in itself if the variable remains within feasible limits after the disturbance. Therefore, the most important indicator is the required value of probability that the random value lies in the specified feasible range.

In the case that the calculation of probabilistic load flow results in critical variables for which the probability to fall within the specified interval is less than the required one, the probability can be increased either by decreasing the mean square deviation of the variable or by shifting its mean value inside the feasible region.

The mean square deviation of the critical variable being also a sensor variable, can be decreased for example by the reinforcement of weak ties [5], [6]. Another possibility is to choose the control actions that decrease the distance between the mean value and median of the distribution density curve on the feasible interval.

Such a criterion is used in the case where variable has the law of distribution other than the normal law, and the approximated probability density curve can be obtained by three or a greater number of moments, using the Gram-Charlie expansion [7].

The probability density curve when approximated on the basis of two moments is symmetrical. This makes it possible to transform the criterion for the selection of controls into the minimization of distance between the mean value of the variable and the center of a feasible interval of its change towards the point of the required probability value. When the constraints on the variable are specified symmetrically with respect to its nominal value the center of the interval is the nominal value of the variable.

In order to choose the controls to provide the required probability for the critical variables to lie within the feasible limits, the method similar to the method of deterministic equivalent [3] is used. In this method successively deterministic and probabilistic problems are solved. Solving the deterministic problem suggests shifting the mean value to the center of the feasible interval but not narrowing this

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interval for each critical variable as it is done in the method of deterministic equivalent.

The variable mean value to be obtained as a result of control can be determined by using the inverse error function. This function allows one to determine the interval $\Delta\varepsilon$ of change in the normally distributed random variable, which makes it possible at a specified value of mean square deviation to provide the required probability that this variable falls within this interval.

The interval $\Delta\varepsilon_i$ is compared to the known feasible interval $\Delta\varepsilon_{if} = 0.5(z_{maxi} - z_{mini})$ of the critical variable z_i change. If $(\Delta\varepsilon_{if} - \Delta\varepsilon_i) > 0$, then for $\mu_{z_i} > (z_{maxi} - \Delta\varepsilon_i)$ the shift of the mean μ_{z_i} of variable z_i will equal $\Delta_{zi} = z_{maxi} - \Delta\varepsilon_i - \mu_{z_i} < 0$, and for $\mu_{zi} < (z_{mini} + \Delta\varepsilon_i)$ the shift will be $\Delta_{zi} = z_{mini} + \Delta\varepsilon_i - \mu_{z_i} > 0$. For $(\Delta\varepsilon_{if} - \Delta\varepsilon_i) < 0$ a conclusion about the impossibility of providing the required probability that the variable lies within the feasible interval and the need to shift the mean value of the variable to the center of the feasible interval is made. In this situation the required shift of the mean μ_{z_i} will be equal to $\Delta_{zi} = z_{mini} + \Delta\varepsilon_{if} - \mu_{z_i}$.

An algorithm for increasing the probability that the variables fall within the feasible limits is iterative, its each k - th iteration contains the following main steps.

1. The deterministic problem is solved to obtain feasible operating conditions of electric power systems subject to

$$W(Z) = 0 \tag{1}$$

$$Z_{min} \le Z \le Z_{max} \tag{2}$$

where (1) – system of equations of nodal power balances, (2)– constraints on the variables Z, that include magnitudes and phases of nodal voltages, active and reactive power flows and independent variables or controls Y, such as active and reactive power of generation and transformation ratios of tap-changing transformers.

Problems (1) and (2) are solved by combining the reduced gradient and quadratic programming methods [3], and if there exists a feasible solution for Z^k and hence for Y^k then the algorithm goes to step 2. Otherwise, the algorithm stops.

2. The probabilistic load flow is calculated, the numerical characteristics of variables and probability of meeting the constraints (2) are determined. If for all the variables the required value of probability is provided, the algorithm stops. Otherwise, the number N_v of critical variables z_j^k is determined for which the required probability value is not provided and an estimate of the shift $\Delta_{z_j}^k$ of its mean which leads to an increase in the probability is calculated. If in the adjacent iterations for each critical variable z_j^k the condition $\left|\Delta_{z_j}^{k+1} - \Delta_{z_j}^k\right| \le \xi$ is met, where ξ – a set small number, the required probability values cannot be reached and the algorithm stops. Otherwise, the algorithm goes to step 3.

3. The deterministic optimization problem is solved to determine the vector of control $Y_* = Y^k + \Delta Y^k$, that provides the minimum of the criterion

$$min \sum_{j=1}^{N_v} \left(z_j(Y) - \left(z_j^k + \Delta_{z_j}^k\right)\right)^2, \tag{3}$$

when the constraints (2) are met. If a solution to the problem is found and criterion (3) equals zero, the algorithm goes to step 4, otherwise, $Y^{k+1} = Y_*$, $k = k+1$, and it goes to step 2.

4. The minimum number of controls ΔY^k is chosen on the basis of the contribution factors method [2]. The information about tracing the flows is used to determine the so called significant controls that affect the critical variable to the greatest extent. This algorithm allows us to determine the paths connecting the critical variable with generator nodes when moving from the node with the critical variable along the electrical network graph in the direction opposite to the orientation of power flows in branches.

The larger the power P_{gn} transferred from generator node g to node n with a critical variable, the greater the impact on the critical variable is produced by the controls (generator power and transformer ratios) included in the path. The power P_{gn} (active or reactive) transmitted from generator node g to load node n in the graph tracing algorithm [2] is determined as

$$P_{gn} = P_g a_{gn} = P_g \overline{P}_n \sum_{j=1}^{l} \prod_{i=1}^{m_j} \overline{P}_i, \tag{4}$$

where a_{gn} – share of generator node power P_g transmitted to load node, $\overline{P}_n$ – relative load at node n, which is determined as a ratio of load power at node to the sum of powers incoming to the node, l – number of paths, connecting the nodes g and n, m_j – number of branches entering the j - th path, and $\overline{P}_i$ – a relative flow at the beginning of branch i, equal to the ratio of power flow in branch P_i to the total power incoming to its initial node.

The significant controls underlie the formation of variants with different number of controls ΔY^k, for each of which the solution to problem (1)–(3) is searched for. When comparing the variants, the variants with the minimum number of controls are chosen. If there are several variants with equal number of controls, then the variant with the minimum active power losses is taken. Then the vector $Y^{k+1} = Y^k + \Delta Y^k$, $k = k + 1$ is determined, and the algorithm goes to step 2.

III. Practical Examples

The electric power system presented in Figure 1, which consists of 14 nodes and 15 ties, is used as a test scheme. The performance of the considered methods for this scheme is illustrated by an example of the nodal voltage magnitude control.

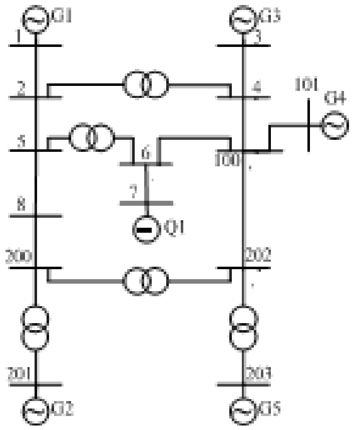

Figure 1 Scheme of a 14-node test network

The initial data on mean values and variances for the loads specified at all the nodes were obtained with the use of the Laplace function. Mean square deviations of nodal powers were assumed to be equal to 12 % of their mean values, which corresponds to 20 % of the load forecast error for a 0.9 probability of random value deviation from the mean.

Figure 2 presents the graphs of the mean square deviations of nodal voltage magnitudes obtained by the linear method and the Monte Carlo method [1]. The graphs show that node 8 is the sensor node.

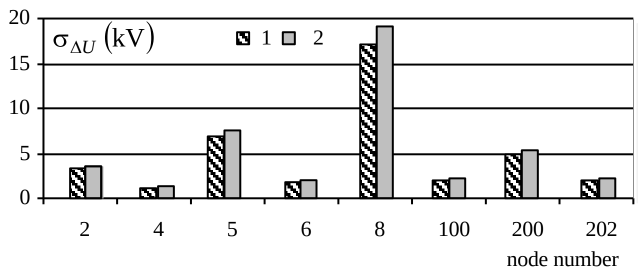

Figure 2 Mean square deviation of voltage magnitudes at the nodes of the test scheme obtained by the linear method – 1, the Monte Carlo method – 2

The conclusion that node 8 is the node with a sensor voltage magnitude can also be made on the basis of the singular analysis technology [5]. For this purpose the nodes can be projected on the plane in coordinates of the first and second singular vectors. The sensor nodes in such a graph will have maximum distance from the origin of coordinates. After interconnecting the nodes by the ties, the network graph projection in coordinates of the first and second right singular vectors is obtained, Figure 3.

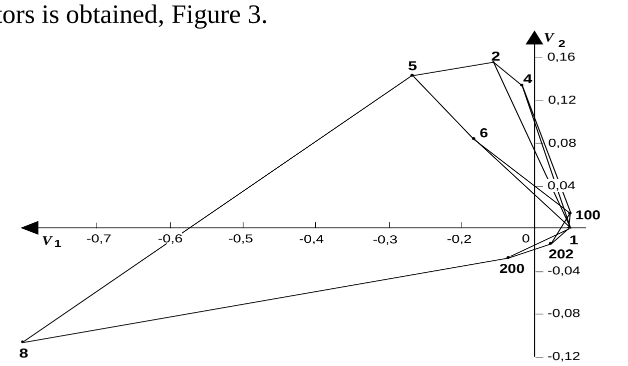

Figure 3. Projection of the network graph in coordinates of the first (**v_1**) and second (**v_2**) right singular vectors that correspond to voltage magnitudes

However, such a technology, unlike the probabilistic load flow, does not allow simultaneous identification of sensor variables and assessment of their possible variation ranges and probabilities that the variables lie within the feasible limits.

Table I contains the mean values μ_U and mean square deviations σ_U of voltage magnitudes obtained by the linear method of probabilistic load flow, differences between the mean values and nominal voltages, and the probabilities p that voltage magnitudes fall within the feasible intervals. The feasible intervals for 500 kV voltages are taken equal to ±30 kV, and for 220 kV – ±25 kV.

TABLE I. Probabilistic Characterictics of Voltage Magnitudes at the Test Network Nodes for the Initial State

Nodes	μ_U **(kV)**	σ_U **(kV)**	$\mu_U - U_{nom}$ **(kV)**	p
2	522.34	3.65	22.34	0.98
4	231.49	1.32	11.49	1.00
5	512.05	6.88	12.05	0.99
6	225.17	1.86	5.17	1.00
8	508.44	17.12	8.44	0.88
100	229.24	2.03	9.24	1.00
200	528.15	4.83	28.15	0.64
202	233.62	2.05	13.62	1.00

a.

If the required value of probability that voltage magnitudes lie within the given intervals should be not less than 0.95, the voltage magnitudes at nodes 200 and 8 can be defined as critical.

Another estimate of critical voltage magnitudes ΔU_i can be represented by the maximum ratio of mean square deviation $\sigma_{\Delta U_i}$ to the feasible variable variation range determined by the proximity of the variable mean to its upper $\overline{\Delta U_i}$ or lower $\underline{\Delta U}_i$ limiting value

$$\phi_{\Delta U_i} = \sigma_{\Delta U_i} \Big/ min\left(\overline{\Delta U_i} - \mu_{\Delta U_i}, \mu_{\Delta U_i} - \underline{\Delta U}_i\right), \qquad (5)$$

Such a possibility is illustrated in Figure 4 which shows the values of components of vector $\phi_{\Delta U}$ and the values of probabilities that voltage magnitudes lie within the feasible limits. Under the initial operating conditions critical node 200 corresponds to the maximum value of $\phi_{\Delta U_i}$ and minimum probability. At the nodes with the 0.98–1.0 probability that voltage magnitudes are within the feasible limits, the values of $\phi_{\Delta U_i}$ do not exceed 0.5.

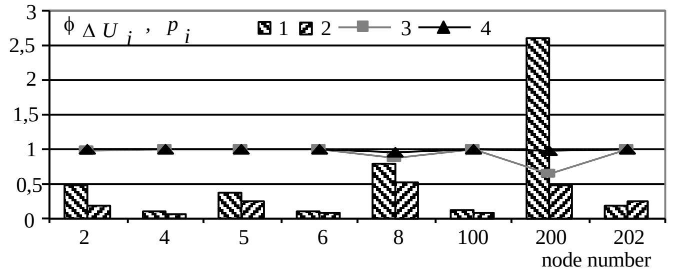

Figure 4. Values of $\phi_{\Delta U_i}$ – 1, 2 and probability p_i that voltage magnitudes are within the feasible limits – 3, 4, for the initial operating conditions 1, 3 and the conditions obtained as a result of the network reinforcement and selection of control actions 2, 4

To provide the required probability, that voltage magnitudes at nodes 8 and 200 fall within the feasible limits, the two methods including the reinforcement of weak ties and selection of control actions to move the mean values of variables to the center of the feasible interval were compared.

Weak ties are the ties, in which the reduction in resistances increases the minimum singular value of the Jacobian matrix, i.e. improves its conditionality and decreases the response of sensor variables to disturbances [5].

For the criterion used to detect weak ties the maximum values of the mean square deviations of changes in the voltage magnitude differences (Figure 5) obtained by the linear method, and the Monte Carlo method are applied [8]. According to these values, ties 5–8 and 8–200 in the test network (Figure 1) are weak. The projection of the network graph (Figure 3) shows that the weak ties are the longest.

At the initial point the mean square deviation of the voltage magnitude at node 8 amounts to $\sigma_{U_8} = 17.12$ kV. This means that with the probability of 0.95 the voltage will be in the interval $\Delta\varepsilon = 33.54$ kV. Since the feasible interval of voltage changes equal to $\Delta\varepsilon_f = 30$ kV, then $(\Delta\varepsilon_{if} - \Delta\varepsilon_i) = (30 - 33.54) = -3.54 < 0$ and the mean should be moved to the center of the feasible interval, hence the mean of the critical variable will be $\mu_{U_8} = 500$ kV. This will make it possible to provide the 0.9203 probability that the voltage falls in the feasible interval.

To provide the 0.99 probability that the voltage of node 200 with mean square deviation equal to $\sigma_{\Delta U_{200}} = 4.83$ kV lies in the feasible interval, the value of the mean should equal $\mu_{U_{200}} = z_{maxi} - \Delta\varepsilon_i = 530 - 11.77 = 518.23$ kV and the shift of the mean will be $\Delta_{U_{200}} = 518.23 - 528.15 = -9.92$ kV.

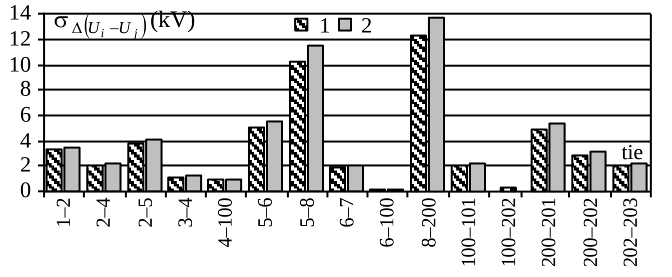

Figure 5. Mean square deviations of voltage magnitude differences in the ties of the scheme obtained by the linear method – 1, the Monte Carlo method – 2

Since the critical variables are represented by voltages, their values are changed by the reactive power sources and transformers.

The test scheme has 11 controls with 6 reactive power sources at nodes 1, 3, 7, 101, 201, 203 and 5 tap-changing transformers in ties 2–4, 5–6, 200–201, 200–202, and 202–203.

The contribution factors method [2] was used to determine the controls that make the required shifts of voltage values at nodes 8 and 200 in the test scheme by investigating the reactive power flows coming to the specified nodes, Figure 6.

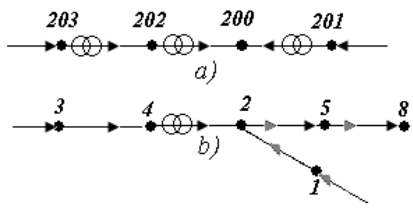

Figure 6. Directions of reactive power flows arriving at nodes 8 and 200 with critical voltage magnitudes

The analysis of the flow directions makes it possible to find the significant controls that include four sources of reactive power at nodes 1, 3, 201 and 203 and four tap-changing transformers 2–4, 200–201, 200–202 and 202–203.

The determined controls are the basis for the calculation of vector ΔY^k with the minimum number of controls that include the reactive power of node 203 and the transformation ratios of transformers 200–201, 200–202 and 2–4.

Figure 7 presents probability that the voltages of nodes 8 and 200 lie in the feasible interval for initial operating condition, after the implementation of determined controls and under simultaneous implementation of the chosen controls and reinforcement of weak ties.

The implementation of control actions related to the change in the source reactive power at node 201 and to the regulation of transformation ratios of transformers, and the obtained shift in the mean values of voltage magnitudes at nodes 8 and 200 make it possible to increase the probability that they lie within the feasible limits.

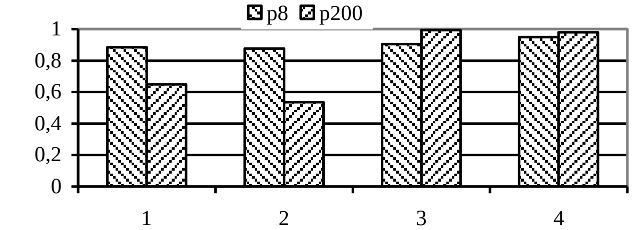

Figure 7. Probability of changes in voltage magnitudes at nodes 8 and 200 for the initial operating conditions (1), operating conditions after the reinforcement of weak ties (2), operating conditions after the implementation of control actions (3), and operating conditions under simultaneous implementation of control actions and reinforcement of weak ties (4)

Owing to the reduction in the mean square deviation and the shift in the mean, simultaneous implementation of control actions and a two-fold increase in the conductivities of weak ties increased the probability that the voltage magnitude at node 8 falls within the feasible limits. Graphs 2 and 4 in Figure 4, which correspond to the values of index $\phi_{\Delta U_i}$ and the probability that voltage magnitudes are within the feasible limits, show that in this case both indices attest to the absence of critical voltage magnitudes.

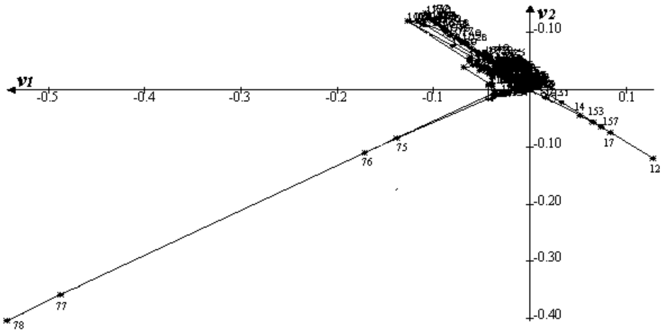

Figure 8. Projection of the real network graph on the plan in coordinates of the first (***v***$_1$) and second (***v***$_2$) right singular vectors that correspond to voltage magnitudes

Let us illustrate the operation of the algorithms with an example of the real electric network consisting of 207 nodes and 224 ties. The projection of the nodes and ties of this network on the plane in coordinates of the first and second singular vectors corresponding to nodal voltage magnitudes is presented in Figure 8. Voltage magnitudes at nodes 77, 78, 12, and 17 (110 kV) are sensor variables. The feasible interval for 110 kV voltages is taken equal to ±10 kV.

Table II shows that the same order of the nodes with sensor voltage magnitudes is obtained by the linear method of probabilistic load flow for the initial and end states. In the initial state the critical variable is only the voltage magnitude at node 12, whose probability of lying within the feasible limits is close to zero. After the shift of the mean, the end state with the 0.95 probability for the critical value is determined. Such a result is obtained using only one control action found by the contribution factors method. The full control vector includes 93 components.

TABLE II. PROBABILISTIC CHARACTERICTICS OF VOLTAGE MAGNITUDES AT THE SENSOR NODES OF REAL NETWORK FOR THE INITIAL (1) AND END (2) STATES

Nodes	σ_U **(kV)**		$m_U - U_{nom}$ **(kV)**		p	
	1	2	1	2	1	2
78	1.267	1.264	–5,69	–5,56	0,999	0,999
77	1,189	1,186	–4,24	–4,12	1	1
12	0,923	0,877	–11,96	–8,55	0,017	0,951
17	0,621	0,594	–7,21	–4,71	1	1
157	0,601	0,573	–7,41	–4,25	1	1

IV. CONCLUSIONS

1. An approach is suggested to search for a solution to the control problem with minimum number of controls on the basis of the data on tracing the power flows.

2. The expressions for the required shift of the mean values of critical variables to the center of the feasible interval were obtained.

3. The criterion for determining the critical variables, which does not require the calculation of the probability, that variable lies in the feasible interval, was proposed.

4. A test and real networks are used as an example to show the efficiency of the proposed approach to reducing the number of controls in order to ensure the required probability of meeting the probabilistic constraints in the case of simultaneous application of the deterministic equivalent and the tracing methods.

The proposed approaches are supposed to be further developed in the studies on the applicability of point methods [9] to calculation of probabilistic characteristics of state variables, design of algorithms for the system security assessment on the basis of the generalized disturbance method [1], and improvement in the methods for solving optimization problems with minimum number of controls for different criteria and calculation cases.

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