Modeling the Dynamics of Isolated Electric Power Systems: Methodology and Algorithms

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Abstract— Most Russia's regions have no centralized electricity supply because of their geographical position. A great number of distributed consumers which can be supplied with electricity only from the autonomous energy sources and problems in the existing decentralized electricity supply systems require that the issues of development and optimization of electricity supply to such consumers be urgently solved. Taking into account variable electricity generation, available energy storage systems, as well as stricter requirements imposed by consumers on power quality and electricity supply reliability, we can say that operating conditions of such systems and their control, including emergency control, represent a difficult problem that needs to be studied. The research is aimed at developing algorithms for the construction of automated systems to control active components of the electrical network. The method applied in the research to describe the object of control is based on the universal approach to the mathematical modelling of nonlinear dynamic system of a black-box type represented by the Volterra polynomials of the N-th degree. This makes it possible for the input and output characteristics of the object to obtain an adequate and fast mathematical description. Results of the computational experiment demonstrate the applicability of the mathematical tool to the control of active components of the intelligent power system.

Keywords— Smart Grid, Volterra polynomials, Power quality, Control systems, Electric power systems, Distributed generation

I. INTRODUCTION

In Russia there are many small isolated areas. A considerable part of Russia's territory, due to its geographical location, is not connected to the centralized power supply. The areas of decentralized energy supply cover almost 60% of Russia's territory and are mainly situated in the north of the country.

The isolated systems can be defined as a small electricity network which serves a single property owner with very simple loads or several communities with complex and interconnected power stations [1].

The isolated electricity supply systems may embrace the following generating capacities: diesel generators; mini hydropower plants; micro-turbines; cogeneration systems; storage systems; power plants on biomass; wind turbines; solar panels, etc. Konstantin Suslov Dmitry Gerasimov Irkutsk National Research Technical University Irkutsk, Russia <u>souslov@istu.edu</u>

The generating capacities are chosen based on the following conditions:

• specific requirements and needs of industrial consumers;

- climatic conditions;
- geographical conditions;
- characteristics of natural and energy resources.

The criterion of climatic conditions, including low temperatures, is the most important for the northern territories of Russia.

The main distinguishing feature of electricity generating facilities based on renewable energy sources is the variability of their parameters. The random deterministic character of the parameters strongly affects the choice of generators.

The issues dealing with the selection of operating conditions, network configuration (in terms of sites for placement of generators), and reliability assessment are considered by us in [2, 3].

II. STATEMENT OF THE PROBLEM

The following facilities will be considered as generators:

- 1. Gas turbine plants.
- 2. Wind turbines.
- 3. Solar panels.

The research does not consider energy storage devices.

The experience of operating a gas turbine plant reveals some serious problems when tuning the automatic control loops, namely:

1. Lack of a comprehensive approach, because energy systems are considered separately from one another.

2. The problem of obtaining common algorithms for control of energy systems, which is related to the complexity of traditional mathematical tools.

We suggest the following approach to solve the above problems. The electricity generating systems are defined by the external structural input-output schemes.

The flow chart for the gas turbine plant is as follows (Fig 1).

The flow chart for the solar panel is as follows (Fig.2).



Fig.1. Flow chart for a single-shaft gas turbine plant (U_E -excitation voltage, U – voltage at the generator outlet, f- network frequency, P- air pressure at the compressor inlet, ω - angular velocity, P_i - pressure at the compressor outlet, F – fuel supplied to the combustion chamber, G- gas flow rate, M_{CC} - gas turbine shaft resistance torque created by compressor, M_{CG} - turbine shaft resistance torque created by electric generator



Fig.2. Flow chart for the solar panel (E_S – solar energy, U_I – voltage at the solar panel outlet).

The flow chart for the wind turbine plant is shown in Fig.3.



Fig.3. Flow chart for the wind turbine plant (V- wind speed, b – attack angle, M_{WT} - engine torque, created by wind turbine)

Each of the generators has their specific features which should be taken into account in designing the automated control system. For example, gas turbine plant makes it possible to completely control input parameters but has quite high inertia. Generation from solar panels is deterministic due to the lack of inertia.

Generation from wind power plants, however, is a vivid example of stochastic operation of generators. At the same time, apart from the random change in the input data such a generation is subject to inertia.

Thus, it is most reasonable to consider wind turbine plant as a generating plant as the most complex component in this isolated system, since it is characterized by both high inertia and stochastic operating conditions.

Figure 4 presents a subsystem of the wind turbine module which is described by a system of algebraic equations, where the input signals are represented by wind speed V, attack angle of turbine blade b, current coordinate of angular velocity ω depending on the shaft resistance torque.

Inertia of the rotating parts in the wind turbine is taken into account by the equation of dynamics

$$\frac{d\omega}{dt} = \frac{M_T - M_C}{J_{\Sigma}},\tag{1}$$

where M_T - turbine generator shaft torque; M_C - resistive torque created by generator; J_{Σ} - total inertia torque.



Fig.4. A subsystem of the wind turbine scheme: C_m - torque coefficient, ρ - air density, V - wind speed, m/s; S - blade-swept area, R - wind wheel radius, m, $C_P = \frac{2P}{\rho V^2 S}$ - coefficient of wind energy use, $Z = \frac{\omega R}{V}$ - specific speed.

Generally speaking, the analysis of dynamic characteristics of wind power unit is based on the methods using differential equations. Most of the researches are devoted to the specification of characteristics of individual components of wind turbine [4, 5], specification of various coefficients [6] or consideration of a mechanical part of the turbine as a n-mass system [7]. In practice, the initial data are known with some error. In this case, as a rule, solutions to the inverse problem turn out to be unstable with respect to an error in the initial data. Therefore, to construct stable methods we use the theory of ill-posed problems [8].

In applications, the nonlinear dynamic input-output systems (objects) are described using the Volterra integropower series [9].

We will name only some of the research areas, in which the Volterra integral power series find their use. These are: modelling of technical systems [10, 11] and electronic devices [12], nonlinear identification of communications channels [13, 14] and visualization systems [15], analysis of non-stationary time series [16], description of automatic feedback control systems [17], modelling and control in agriculture [18].

III. MATHEMATICAL MODEL OF A CLOSED-LOOP CONTROL SYSTEM

We suggest an approach which involves a mathematical model of a closed-loop control system based on the finite interval of the Volterra integro-power series:

$$y(t) = \sum_{n=1}^{N} \sum_{1 \le i_1 \le \dots \le i_n \le p} f_{i_1,\dots,i_n}(t), t \in [0,T],$$
(2)

where

$$f_{i_1,\dots,i_n}(t) = \int_0^t \dots \int_0^t K_{i_1,\dots,i_n}(t,s_1,\dots,s_n) \prod_{j=1}^n x_{i_j}(s_j) ds_j, t \in [0,T], \quad (3)$$

t has a physical sense of time, input signal $x(t) = (x_1(t), ..., x_p(t))$ is a *p*-dimensional vector-function of time, output of the system y(t) – a scalar function of time, where y(0) = 0, $y'(t) \in C_{[0,T]}$. Based on the physical sense of the model (2), (3), the Volterra kernels $K_{i_1,...,i_n}$ in (3) are transfer functions and are symmetric with respect to the variables which correspond to coinciding indices.

To construct an integral model in the form (2), (3) means to restore multidimensional transient characteristics of the nonlinear dynamic system $K_{i_1,...,i_n}$. Currently, there are quite many methods developed to determine the dynamic characteristics [19, 20]. The most widely used approach is presented in [21]. It suggests setting a multiparametric family of test signals consisting of a combination of Dirac delta functions to recover the Volterra kernels. However, such an approach has limited application [22].

The identification methods used by the authors [23, 24] are based on the specification of a family of test signals. In this case, the identification problem is reduced to solving the linear Volterra integral equations of the first kind which have explicit inverse formulas.

Let us consider the stabilization (regulation) problem connected with the search for control action $x_1(t)$ which maintains the input signal y(t) at a specified level y^* . Such a statement appears in relation to the problems of automatic control of technical facilities. Assume that the functions $K_{i_1,...,i_n}$ ($K_1(t,t) \neq 0 \quad \forall t \in [0,T]$) and inputs $x_i(t)$, $i = \overline{2, p}$, in (3) are known.

As a reference dynamic system, we will consider a mathematical model of horizontal-axis wind turbine represented using the techniques [25-27].

We will consider the case for N = 2 in (2), as it is the most interesting for applications. In the case of non-stationary dynamic system instead of (2), (3) we have

$$\sum_{i=1}^{2} V_{1,i} x_{i} + \sum_{i=1}^{2} V_{2,i} x_{i}^{2} + V_{2,12}(x_{1}, x_{2}) = y(t), t \in [0,T], \quad (4)$$

$$V_{1,i}x_{i} \equiv \int_{0}^{t} K_{i}(t,s)x_{i}(s)ds , \qquad (5)$$

$$V_{2,i}x_i^2 \equiv \int_{0}^{t} \int_{0}^{t} K_{ii}(t,s_1,s_2)x_i(s_1)x_i(s_2)ds_1ds_2 , \qquad (6)$$

$$V_{2,12}(x_i, x_j) \equiv \int_{0}^{t} \int_{0}^{t} K_{12}(t, s_1, s_2) x_1(s_1) x_2(s_2) ds_1 ds_2.$$
(7)

In this case equation (4) is a polynomial Volterra equation of the first kind, and its continuous solution is of a local character. The papers [24, 28] present the results in the field of theory and numerical methods for the construction of continuous solutions to the polynomial equations (at N = 2,3in (2)) for the case of stationary dynamic systems with scalar input x(t). In [29] consideration is given to a numerical scheme of solving the polynomial equation (4)-(7) provided there is no feedback.

In [30] the approach proposed by us was for the first time applied to describe the nonlinear dynamics of the output signal $y(t) \equiv \Delta \omega_T(t) = \omega_T(t) - \omega_{T_0}$, $\omega_{T_0} = 23.5$ (rad/s) of the wind turbine with input actions $x_1(t) \equiv \Delta b(t) = b(t) - b_0$, $x_2(t) \equiv \Delta V(t) = V(t) - V_0$ of arbitrary form with the use of (4) where

$$V_{1,i}x_i = \int_0^t K_i(t-s)x_i(s)ds, \qquad (8)$$

$$V_{2,i}x_i^2 \equiv \int_0^t \int_0^t K_{ii}(t-s_1,t-s_2)x_i(s_1)x_i(s_2)ds_1ds_2, \quad (9)$$

$$V_{2,12}(x_i, x_j) \equiv \int_{0}^{t} \int_{0}^{t} K_{12}(t - s_1, t - s_2) x_1(s_1) x_2(s_2) ds_1 ds_2.$$
(10)

Transient responses $K_1, K_2, K_{11}, K_{22}, K_{12}$ from (8)-(10) were restored based on the outputs of the reference model (1) of the wind turbine plant, whose flow chart is presented in Figs.3, 4. The applied identification methodology [23, 24] is based on special test signals represented by special linear combinations of Heaviside functions

$$e(t) = \begin{cases} 0, \ t < 0, \\ 1, \ t \ge 0, \end{cases}$$

with amplitudes α . To assess the accuracy of the integral model (4), (8)-(10) the outputs $\Delta \omega_T(t)$ to the input signals $\Delta b(t)$, $\Delta V(t)$ of arbitrary form were compared to the outputs of the reference model (1).

Table I presents relative and absolute errors of modeling the output of the system to the input signals:

1.
$$\Delta b(t) = -10e(t), \quad \Delta V(t) = 5e(t),$$

2. $\Delta b(t) = -20e(t), \quad \Delta V(t) = 10e(t),$
3. $\Delta b(t) = -10e(t), \quad \Delta V(t) = 5(e(t) - e(t - 8)),$
4. $\Delta b(t) = -10e(t), \quad \Delta V(t) = 5(e(t) - e(t - 12)),$
5. $\Delta b(t) = -10(e(t) - e(t - 1)), \quad \Delta V(t) = 5e(t),$
6. $\Delta b(t) = -10(e(t) - e(t - 3)), \quad \Delta V(t) = 5e(t),$
7. $\Delta b(t) = -20(e(t) - e(t - 14)), \quad \Delta V(t) = 10e(t),$
8. $\Delta b(t) = -20(e(t) - e(t - 4)), \quad \Delta V(t) = 10e(t),$
9. $\Delta b(t) = -20e(t), \quad \Delta V(t) = 10(e(t) - e(t - 1)),$
10. $\Delta b(t) = -20e(t), \quad \Delta V(t) = 10(e(t) - e(t - 7)),$

using the integral models (13) and (14).

TABLE I. RELATIVE AND ABSOLUTE ERRORS

Examples of the input signals	\mathcal{E}_1	\mathcal{E}_2	E ₃	\mathcal{E}_4	\mathcal{E}_5
1	0.00	0.000	0.00	0.00	0.00
2	0.00	0.000	0.00	0.00	0.00
3	1.89	0.008	6.26	0.02	0.00
4	1.89	0.068	8.04	0.29	0.00
5	1.17	0.003	4.98	0.01	0.00
6	2.33	0.009	9.91	0.04	0.00
7	1.39	0.330	5.91	1.40	0.00
8	1.28	0.006	5.45	0.03	0.00
9	1.29	0.007	5.49	0.03	0.00
10	2.41	0.012	10.25	0.05	0.00

The notations used in Table I:

$$\begin{split} \varepsilon_1 &= \max_{1 \leq i \leq T} |\Delta \omega_T(t_i) - y_2(t_i)| \text{ (rad/s)}, \\ \varepsilon_2 &= |\Delta \omega_T(T) - y_2(T)| \text{ (rad/s)}, \\ \varepsilon_3 &= \frac{\varepsilon_1}{\omega_{T_0}} \cdot 100\% \text{ (in \%)}, \ \varepsilon_4 &= \frac{\varepsilon_2}{\omega_{T_0}} \cdot 100\% \text{ (in \%)}, \\ \varepsilon_5 &= \max_{1 \leq i \leq T} |\Delta \omega_T(t_i) - y_3(t_i)| \text{ (rad/s)}, \\ b_0 &= 20 \text{ (deg)}, \ V_0 &= 5 \text{ (m/s)}, \ t_i &= i \cdot h, \ i = \overline{1, 20}, \\ \omega_{T_0} &= 23.5 \text{ (rad/s)}, \ h = 1 \text{ (s)}, \ T = 20 \text{ (s)}. \end{split}$$

It should be noted that the model built using only one group of signals cannot be considered equally suitable for the calculation in the entire range of admissible changes in the input signals. In order to improve the accuracy of modelling we introduced initial reference conditions $V_0 = 8$; 10 (m/s),

 $b_0 = 0;10;20$ (deg) for which the models of form (4) were constructed.

In this case, all the deviations of input and output parameters which are assumed as actuating signals and responses to them were calculated with respect to the chosen reference conditions.

Note, that in practice the a priori information about whether or not the dynamic system to be modeled is stationary, is unknown.

We will illustrate the results of an a posteriori analysis. In particular, we will consider a two-dimensional response of the system to the test input $x_{\omega_1}(t) = e(t) - e(t - \omega_1)$. In the case of $K'_{1_t} \neq -K'_{1_{\omega_1}}$, $K_1(t, \omega_1) \equiv K_1(t - \omega_1)$ and, therefore, the system is stationary.

Figure 6 presents the graphs of $\mathcal{E} = |K'_{1t}| - |K'_{1\omega_1}|$ at $t = \omega_1$. The graph of \mathcal{E}_1 is obtained in the case where the Volterra kernels from (5) are constructed for $V_0 = 8$ (m/s), and $\mathcal{E}_2 - V_0 = 10$ (m/s). Figure 7 shows the graphs of $\mathcal{E} = |K'_{2t}| - |K'_{2\omega_1}|$ at $t = \omega_1$. The graph \mathcal{E}_1 is obtained in the case where the Volterra kernels from (5) are constructed for $b_0 = 10$ (deg), \mathcal{E}_2 - for $b_0 = 20$ (deg), \mathcal{E}_3 - for $b_0 = 0$ (deg).

It is obvious (Figs. 6 and 7) that the studied dynamic system of the wind power plant is non-stationary in the considered range of input parameters. Therefore, to describe the nonlinear dynamics it is preferable to use model (4)-(7) rather than the simplified model (4), (8)-(10). The algorithm of constructing (4)-(7) was implemented by using the required solvability conditions of respective multi-dimensional Volterra integral equations of the first kind.





IV. APPLICATION OF A PRODUCT INTEGRATION METHOD TO CONSTRUCT A CUBIC VOLTERRA POLYNOMIAL

A disadvantage of the approach in [23] lies in the fact that the multidimensional Volterra integral equations of the first kind, to which the identification of K_m is reduced, have solutions in the needed classes of functions under rather burdensome conditions. At the same time, knowledge of kernels themselves is redundant when we need to forecast a response of the system to any external disturbance. With a sufficiently small mesh step h it is possible to approximate the multidimensional convolutions in (3) according to the product integration method [31] and pass from (2), (3) to the expression

$$y(ih) = \sum_{\nu=1}^{N} \sum_{1 \le i_1 \le \dots \le i_{\nu} \le m} f_{i_1,\dots,i_{\nu}}(ih), t_i = ih,$$
(11)

$$f_{i_1,\dots,i_\nu}(ih) = \sum_{i_1,\dots,i_m=1}^{i} g_{i_1,\dots,i_m} \prod_{k=1}^{m} x\left(\left(i - i_k + \frac{1}{2}\right)h\right), \quad (12)$$

$$g_{i_1,...,i_m} = \int_{(i_1-1)h}^{i_1h} \dots \int_{(i_m-1)h}^{i_mh} K_m(s_1,...,s_m) ds_1...ds_m, \quad (13)$$

where $i = \overline{1,n}$, nh = T, $i_1, \dots, i_m = \overline{1,n}$. In this case it is enough to be able to restore expression (12), (13). For the scalar case this can be done using the test disturbances from [23] or using any other way, on the basis of special families of piecewise-constant test inputs with a width h. Such an approach is described in [32] as applied to a scalar case.

The complexity in the identification of models with vector input consists in finding asymmetric and partially symmetric integrals of the kernels that reflect the contribution of two and more different inputs. The technique of finding the asymmetric integrals of the kernels implies creating a set of the dynamic system responses to the test disturbances of a certain kind. This allows us to have a necessary amount of equations and obtain inverse formulas of the sought integrals. In order to identify partially symmetric kernels in the set

(12), (13) it is necessary to collect $n^3 + 2n^2$ linearly independent equations of form (11), which we will gather using the following inputs

$$\begin{aligned} x_{i}^{\alpha_{\lambda}}(t_{i}) &= \alpha_{\lambda} \left(e(t_{i}) - e(t_{i-1}) \right), \\ x_{i,j,k}^{\alpha_{\mu}}(t_{i}) &= \alpha_{\mu} \left(e(t_{i-j}) - e(t_{i-j-1}) \right) + \\ &+ \alpha_{\mu} \left(e(t_{i-k}) - e(t_{i-k-1}) \right), \\ x_{i}^{-\alpha_{\mu}}(t_{i}) &= -\alpha_{\mu} \left(e(t_{i}) - e(t_{i-1}) \right) \end{aligned}$$

and

$$\begin{split} x_{i}^{\alpha_{\mu}}(t_{i}) &= \alpha_{\mu} \left(e(t_{i}) - e(t_{i-1}) \right), \\ x_{i,j,k}^{\alpha_{\lambda}}(t_{i}) &= \alpha_{\lambda} \left(e(t_{i-j}) - e(t_{i-j-1}) \right) + \\ &+ \alpha_{\lambda} \left(e(t_{i-k}) - e(t_{i-k-1}) \right), \\ x_{i}^{-\alpha_{\lambda}}(t_{i}) &= -\alpha_{\lambda} \left(e(t_{i}) - e(t_{i-1}) \right), \end{split}$$

 $i, j, k = \overline{1, n}$. After solving this system we obtain the inversion formulas, for example:

$$p_{i,i-j}^{\alpha_{\lambda}\alpha_{\mu}} = \frac{-y_{iij}^{-\alpha_{\lambda}-\alpha_{\lambda}\alpha_{\mu}} - y_{iij}^{\alpha_{\lambda}\alpha_{\lambda}-\alpha_{\mu}} + 2\alpha_{\lambda}^{2}p_{ii}^{\alpha_{\lambda}\alpha_{\lambda}} + 2\alpha_{\mu}^{2}p_{jj}^{\alpha_{\mu}\alpha_{\mu}}}{2\alpha_{\lambda}\alpha_{\mu}}.$$

When using the inputs

$$x_{i,j}^{\alpha_{\lambda}}(t_{i}) = \alpha_{\lambda} \left(e(t_{i}) - e(t_{i-1}) \right),$$

$$x_{i,j}^{\alpha_{\mu}}(t_{i}) = \alpha_{\mu} \left(e(t_{i-j}) - e(t_{i-j-1}) \right),$$

$$x_{i,k}^{\alpha_{\nu}}(t_{i}) = \alpha_{\nu} \left(e(t_{i-k}) - e(t_{i-k-1}) \right),$$

 $\lambda = \overline{1,3}$, $\mu < \nu$, we find the asymmetric kernels, for example:

1

$$q_{i,i-j,i-k}^{\alpha_{1}\alpha_{2}\alpha_{3}} = \frac{1}{\alpha_{1}\alpha_{2}\alpha_{3}} \left(y_{ijk}^{\alpha_{1}\alpha_{2}\alpha_{3}} - m_{i}^{\alpha_{1}} - m_{i-j}^{\alpha_{2}} - m_{i-k}^{\alpha_{3}} - p_{ii}^{\alpha_{1}\alpha_{1}} - p_{i-j,i-j}^{\alpha_{2}\alpha_{2}} - p_{i-k,i-k}^{\alpha_{3}\alpha_{3}} - p_{i,i-j}^{\alpha_{1}\alpha_{2}} - p_{i,i-k}^{\alpha_{1}\alpha_{3}} - p_{i,i-k}^{\alpha_{2}\alpha_{2}} - q_{ii,i-j}^{\alpha_{1}\alpha_{1}} - q_{i-j,i-j}^{\alpha_{2}\alpha_{2}\alpha_{2}} - q_{i-k,i-k,i-k}^{\alpha_{1}\alpha_{1}\alpha_{1}} - q_{i-j,i-j,i-j}^{\alpha_{2}\alpha_{2}\alpha_{3}} - q_{i-k,i-k,i-k,i-k}^{\alpha_{1}\alpha_{2}\alpha_{2}} - q_{i,i-j,i-j}^{\alpha_{1}\alpha_{3}\alpha_{3}} - q_{i,i-j,i-j}^{\alpha_{2}\alpha_{2}\alpha_{3}} - q_{i,i-j,i-j}^{\alpha_{1}\alpha_{3}\alpha_{3}} - q_{i,i-j,i-j}^{\alpha_{1}\alpha_{2}\alpha_{2}} - q_{i,i-k,i-k}^{\alpha_{1}\alpha_{2}\alpha_{2}} - q_{i,i-k,i-k}^{\alpha_{1}\alpha_{2}\alpha_{2}} - q_{i,i-k,i-k}^{\alpha_{1}\alpha_{2}\alpha_{2}} - q_{i,i-k,i-k}^{\alpha_{1}\alpha_{2}\alpha_{2}} - q_{i,i-k,i-k}^{\alpha_{1}\alpha_{2}\alpha_{2}} - q_{i,i-j,i-j}^{\alpha_{1}\alpha_{3}\alpha_{3}} - q_{i,i-j,i-j}^{\alpha_{2}\alpha_{2}\alpha_{3}} - q_{i,i-j,i-j}^{\alpha_{1}\alpha_{3}\alpha_{3}} - q_{i,i-j,i-j}^{\alpha_{2}\alpha_{2}\alpha_{3}} - q_{i,i-k,i-k}^{\alpha_{1}\alpha_{2}\alpha_{2}} - q_{i,i-k,i-k}^{\alpha_{1}\alpha_{2}\alpha_{2}} - q_{i,i-k,i-k}^{\alpha_{1}\alpha_{2}\alpha_{2}} - q_{i,i-k,i-k}^{\alpha_{2}\alpha_{2}\alpha_{3}} - q_{i,i-k,i-k}^{\alpha_{1}\alpha_{2}\alpha_{2}} - q_{i,i-k,i-k}^{\alpha_{1}\alpha_{2}\alpha_{2}} - q_{i,i-k,i-k}^{\alpha_{2}\alpha_{2}\alpha_{3}} - q_{i,i-j,i-j}^{\alpha_{2}\alpha_{2}\alpha_{3}} - q_{i,i-k,i-k}^{\alpha_{2}\alpha_{3}\alpha_{3}} - q_{i,i-j,i-j}^{\alpha_{2}\alpha_{2}\alpha_{3}} - q_{i,i-k,i-k}^{\alpha_{2}\alpha_{2}\alpha_{3}} - q_{i,i-k,i-k}^{\alpha_{2}\alpha_{3}\alpha_{3}} - q_{i,i-k,i-k}^{\alpha_{2}$$

To test the derived formulas, a specified mathematical "test" model was used to conduct numerical experiments. The experiments demonstrate the efficiency of both models.

V. CONCLUSIONS

The presented results of the mathematical modeling using the finite interval of the integro-power Volterra series were for the first time applied to describe the dynamics of the horizontal-axis wind turbine. The technique was developed to construct the integral model and technically implement the high-speed system of control. A computational experiment aimed at constructing the integral models of the wind power unit was done.

The results of the computational experiment demonstrate the applicability of this mathematical tool to the control of active components of the electric power system.

We plan to use a new algorithm for identifying Volterra polynomials to improve the accuracy of simulation. Algorithm is based on the product integration method. The computer modeling was carried out using the author's software created in Matlab.

Further it is planned to apply this approach to the research into complex dynamic systems which contain an arbitrarily large amount of components of the active-adaptive isolated system. This mathematical apparatus will allow us to solve the problem of automatic control at a higher technical level.

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