Scalable Heuristics for Planning, Placement and Sizing of Flexible AC Transmission System Devices

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***Abstract*—Aiming to relieve transmission grid congestion and improve or extend feasibility domain of the operations, we build optimization heuristics, generalizing standard AC Optimal Power Flow (OPF), for placement and sizing of Flexible Alternating Current Transmission System (FACTS) devices of the Series Compensation (SC) and Static VAR Compensation (SVC) type. One use of these devices is in resolving the case when the AC OPF solution does not exist because of congestion. Another application is developing a long-term investment strategy for placement and sizing of the SC and SVC devices to reduce operational cost and improve power system operation. SC and SVC devices are represented by modification of the transmission line inductances and reactive power nodal corrections respectively. We find one placement and sizing of FACTs devices for multiple scenarios and optimal settings for each scenario simultaneously. Our solution of the nonlinear and nonconvex generalized AC-OPF consists of building a convergent sequence of convex optimizations containing only linear constraints and shows good computational scaling to larger systems. The approach is illustrated on single- and multi-scenario examples of the Matpower case-30 model.**

***Index Terms*—Series Compensation Devices, Static VAR Compensation Devices, Non-convex Optimization, Optimal Power Grid reinforcement, Optimal Investment Planning**

I. Introduction

Power grids require flexibility to meet new operational challenges related to grid expansion [1], increasing penetration of renewables [2], [3] and generation retirement [4]. In addition to traditional ways to balance AC-flows through generation dispatch, a number of new technological solutions are now available for controls. In particular, installation of the so-called Flexible Alternating Current Transmission System (FACTS) adds an important new option to the mix of other available control options, see e.g. [5], [6], [7], [8] and references therein.

Serial Compensation (SC) and Static VAR Compensation (SVC) are FACTS devices of new type [9], [10], [11], [12] which generally represent a way to compensate lines or loads respectively. Main effect of an SC device consists in modifying line inductance, while an SVC device injects or consume reactive power.

Planning the installation of new FACTS devices with sufficient capacity and flexibility to meet requirements of multiple demand scenarios, e.g. accounting for seasonal variations and growth of demands is the challenging optimization problem discussed in the past by many other authors, e.g. [5], [6], [7], [8], and also addressed in this manuscript.

Our project has started from resolving the challenge within the paradigm of DC approximation in [13], [14]. In this manuscript we develop the optimization framework of FACTS placement, sizing and operational optimality which accounts for the most general AC case and works with multiple scenarios. We pose an optimization problem that extends the standard AC OPF with new optimization degrees of freedom related to the flexibility in line inductances and reactive power corrections provided by SC and SVC devices. The problem is stated as a network optimization problem, which is generally non-convex and nonlinear. Then we build efficient optimization heuristics which constructs a convergent and carefully controlled sequence of convex optimizations with linear constraints. These convex optimizations are solved efficiently with modern on-the-shelf software (like Gurobi [15] or CPLEX [16]). This brief manuscript presents first results of the new AC-based multi-scenario solver that optimizes over both standard generation dispatch and FACTS-related degrees of freedom. We mainly focus on describing the general logic and some technical details of the algorithm. We also illustrate performance of the algorithm on a popular 30 node model from Matpower [17].

The material in the manuscript is organized as follows. We set the stage in Section II by introducing notations. The problem is stated as an optimization in Section III. We describe the solution algorithm in Section IV, illustrate performance of the algorithm/solver(s) in Section V and conclude in Section VI.

II. Notation

In this Section we introduce/remind terminology and nomenclature. We begin by describing the network setting and introducing notation.

- Layout of the power transmission network, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ and $\mathcal{E}$ represent the set of nodes and edges of the network/graph, with line characteristics such as inductances, resistances and shunt capacitances is known.

- List of projected scenarios, i.e. different load configurations, $a = 1, \cdots, N$. The scenarios may include sampled (typical) configurations and/or contingency (rare) configurations projected for different level of loading.
- Each scenario is characterized by:
  - Occurrence probability
  - State of the network – energized network is the subgraph of the complete network
  - State of generators – list of generators on-line
  - Configuration of loads

List of fixed parameters characterizing scenario a is as follows:

- $T^{(a)}$ - temporal rate (frequency) of scenario occurrence
- $\mathcal{G}^{(a)} = (\mathcal{V}^{(a)}, \mathcal{E}^{(a)}) \subseteq \mathcal{G}$ - energized subgraph of the full network, $\mathcal{G}$
- $x_0^{(a)} = (x_{ij;0}^{(a)} | \{i,j\} \in \mathcal{V}^{(a)})$ - vector of initial inductances of energized lines
- $r^{(a)} = (r_{ij}^{(a)} | \{i,j\} \in \mathcal{E}^{(a)}$ - vector of resistances of lines
- $b^{(a)} = (b_{ij}^{(a)} | \{i,j\} \in \mathcal{E}^{(a)}$ - vector of shunt capacitances of lines
- $P^{(a)}_{min(max)-gen} = (P^{(a)}_{i;min(max)-gen} | i \in \mathcal{V}_g^{(a)} \subset \mathcal{V}^{(a)})$ - vectors of minimum (maximum) active power outputs of energized generators
- $Q^{(a)}_{min(max)-gen} = (Q^{(a)}_{i;min(max)-gen} | i \in \mathcal{V}_g^{(a)})$ - vectors of minimum (maximum) reactive power outputs of energized generators
- $P^{(a)}_{load} = (P^{(a)}_{i;load} | i \in \mathcal{V}_l^{(a)} \subset \mathcal{V}^{(a)})$ - vector of active power consumptions at loads
- $v^{(a)}_{\min(\max)} = (v^{(a)}_{i;\min(\max)} | i \in \mathcal{V}^{(a)})$ - vectors of minimum (maximum) allowed voltages
- $S^{(a)}_{\max} = (S^{a}_{ij,\max} | \{i,j\} \in \mathcal{E}^{(a)} \subset \mathcal{E})$ - vector of the apparent power limits of energized lines

The following are scenario-indexed degrees of freedom which are optimized over:

- $x^{(a)} = (x_{ij}^{(a)} | \{i,j\} \in \mathcal{V}^{(a)})$ - vector of line inductances (modified by SC devices)
- $Q^{(a)} = (Q_i^{(a)} | i \in \mathcal{V}^{(a)})$ - vector of reactive power injection/consumption at loads and generators (modified by SVC devices)
- $P_g^{(a)} = (P_{i;g}^{(a)} | i \in \mathcal{V}_g^{(a)})$ - vector of active power injections at the generators (operational cost for each scenario is cost of active power generation)
- $v^{(a)} = (v_i^{(a)} | i \in \mathcal{V}^{(a)})$ - vector of operational voltages
- $\theta^{(a)} = (\theta_i^{(a)} | i \in \mathcal{V}^{(a)})$ - vector of operational phases

Cost of the device placement and related service period:

- C_{SC} - SC capacity placement cost (per 1 Ohm)
- C_{SVC} - SVC capacity placement cost (per 1 MVar)
- N_y - service period of the system

Finally, global (i.e. scenario independent) optimization degrees of freedom are:

- $\overline{\triangle x} = (\overline{\triangle x}_{ij} | \{i,j\} \in \mathcal{E})$ - vector of SC device capacities (positive values, allowed up and down regulated intervals are assumed equal)
- $\overline{\triangle Q} = (\overline{\triangle Q}_i | i \in \mathcal{V}_l)$ - vector of SVC device capacities (positive values, regulated up and down intervals are assumed equal)

To account for operational flexibility of the devices we use as optimization variables scenario independent capacities and independently actual correction values for the devices contained within the capacity limits. We utilize the standard π-model for lines, however without tap changers and phase shifters for simplicity (they can be easily added to the model).

III. Problem Formulation

The problem is to place and size FACTS devices of SC and SVC types in a way that the combination of the cost of the upgrade and the cost of operations (load configurations), will be minimized:

$$\min_{\overline{\triangle x}, \overline{\triangle Q}; \text{state}^{(a)}, \forall a} \text{COST}\left(\overline{\triangle x}, \overline{\triangle Q}; \text{state}^{(a)}\right) \quad (1)$$

$$\text{COST} \doteq (C_{SC} \sum_{\{i,j\} \in \mathcal{E}} \overline{\triangle x_{ij}} + C_{SVC} \sum_{i \in \mathcal{V}_l} \overline{\triangle Q_i}$$
$$+ N_y \sum_{a=1..N} T_a * C_a(P^{(a)})) \quad (2)$$
$$\text{state}^{(a)} \doteq (x^{(a)}, v^{(a)}, \theta^{(a)}, Q^{(a)}, P^{(a)}), \ \forall a$$
$$x^{(a)} = x_0^{(a)} + \triangle x^{(a)} \quad \forall a$$
$$Q^{(a)}_{load} = Q^{(a)}_{load-0} + \triangle Q^{(a)}_{load}, \ \forall a$$
$$-\overline{\triangle x} \le \triangle x^{(a)} \le \overline{\triangle x}, \ \forall a$$
$$-\overline{\triangle Q} \le \triangle Q^{(a)}_{load} \le \overline{\triangle Q}, \ \forall a$$
$$v^{(a)}_{\min} \le v^{(a)} \le v^{(a)}_{\max}, \ \forall a$$
$$Q^{(a)}_{min-gen} \le Q^{(a)}_{gen} \le Q^{(a)}_{max-gen}, \ \forall a$$
$$P^{(a)}_{min-gen} \le P^{(a)}_{gen} \le P^{(a)}_{max-gen}, \ \forall a$$
$$\sqrt{(P_{ij}^{(a)})^2 + (Q_{ij}^{(a)})^2} \le S^{(a)}_{\max} \quad \forall a; \ \forall \{i,j\} \in \mathcal{E}^{(a)}$$
$$P_i^{(a)} + iQ_i^{(a)} = \sum_{j:\{i,j\} \in \mathcal{E}^{(a)}} \left(S_{ij}^{(a)}\right), \ \forall i \in \mathcal{V}^{(a)}, \ \forall a$$

where all the inequalities containing vectors are considered component-wise; $C_a(P^{(a)})$ stands for the function representing the cost of generation for scenario a, and $\forall a$ is a shortcut for, $\forall a = 1, \cdots, N$. The objective function (2) represents capital investment cost of the installation of two types of FACTS devices (taking linear in the installation capacities and thus promoting sparseness, see [13], [14] for related discussion) plus operational cost summed over all the scenarios with their probabilities taken into account and multiplied by the number of years (service period). The optimization constraints above have the following meaning:

- state for each scenario is defined by vectors of line inductances, voltages, phases, active and reactive power injections at nodes
- actual line inductance is equal to its initial value plus SC correction adjusted to a scenario, however maintained within the installed capacity bounds
- actual reactive power demand for a load is equal to its initial value plus SVC adjusted to a scenario, , however maintained within the installed capacity bounds

- limits for reactive power generation
- limits for active power generation
- line thermal limits
- active and reactive power balance at nodes

Thermal and power balance constraints are non-linear and non-convex. In order to resolve this complication we develop the iterative heuristic approach to solve Eq. (1) described in the next Section.

IV. Optimization Algorithm

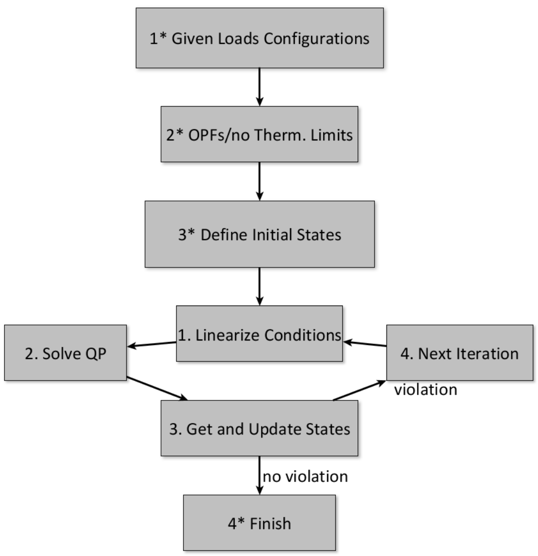

Fig. 1. Flowchart of our iterative algorithm.

The idea of the algorithm is to proceed sequentially. At each step within the sequence we linearize the constraints around a current state (found at the preceding step) and then use a standard convex optimization solver to evaluate the resulting quadratic programming (QP) optimization (for the generation cost modelled as a quadratic function) with linear constraints. We also control each step so that the resulting correction to the current state is sufficiently small to justify the linearization. The algorithm is terminated when preset target precision/accuracy is reached. Flowchart of the algorithm is shown in Fig. 1, where the underlying details are as follows:

1* Each load configuration (each scenario) is given (see next Section for discussion on how we generate the scenarios).

2* Here for each scenario we solve OPF with thermal limits removed (standard OPF can be infeasible for some load configurations and this could be resolved with the FACTS placement).

3* Initial state for each scenario is taken from step 2*.

1. Linearization of the thermal and power balance constraint over the current state for each scenario is applied.
2. QP is evaluated. Here we artificially restrict change of reactive power injections on generators. (The restriction is caused by empirical observation that for a system with multiple alternative loops for power flows and reactive power assumed injected/consumed at no cost, there may be multiple solutions with close costs and similar active power generation, however showing rather distinct profiles of reactive power injection/consumption.)
3. This step assumes that a feasible solution of the preceding optimization problem is found. The solution outputs active powers and voltages at the generators, active and reactive powers at loads which we then use to run AC Power Flow (AC-PF) solver (in our experiments we use Mathpower solver) to update the remaining parameters of the current state.
4. One arrives at this step if some constraints of the preceding optimization problem are violated. One checks if the number of iterations is still less than the maximum allowed and then proceed to the next iteration, or exit (declaring infeasibility) otherwise.

In this scheme linearization of the constraints is straightforward. We consider apparent power squared flowing through the line $\{i,j\}$ under the scenario, a, $(P_{ij}^{(a)})^2 + (Q_{ij}^{(a)})^2$, as well as real and reactive powers injected/consumed at a node, i, under scenario, a, $P_i^{(a)}$ and $Q_i^{(a)}$, as functions of the state variables (x, v, θ), and then linearize around the current state by computing partial derivatives over state variables explicitly.

V. Simulations and Results

We have first developed a single-scenario solver following the scheme of Fig. 1 and tested it extensively. In particular, the solver was validated against the Matpower OPF solver and it was also verified that the solver is producing sensible results for dependence of the optimal cost and configuration on the exogenous parameters, e.g. on duration of the period of performance. (The investment term in the optimum cost starts to dominate the operational term in the cost only if the duration is sufficiently long.) Then to actually solve Eq. (1) we have built the multi-scenario solver upon the experience gained. In the remaining part of this Section we illustrate operations and abilities of our approach on example of the Matpower 30-node case.

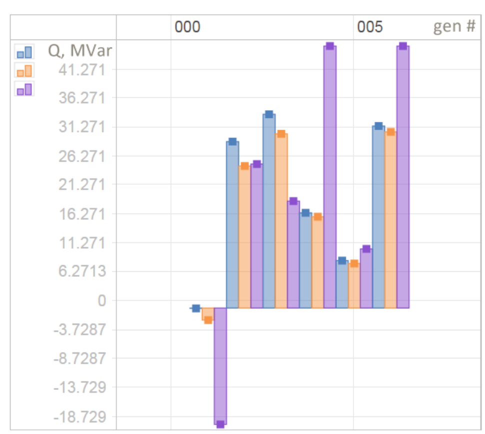

Fig. 2. Comparison of the reactive power dispatch (Q, MVar) at six generators of the 30-bus system for three different configurations marked as blue, orange and purple. Blue bars - initial state of the algorithm (step 3*). Orange bars shows final stage of our algorithm - feasible and minimizing the overall cost. Purple shows outputs of the standard OPF run (marked as infeasible by solver).

First, we experiment with an infeasible single load configuration, i.e. configuration without FACTS corrections for which there exists no generation dispatch meeting all the generation and line-thermal limits. This load case was created by overloading the feasible base case of the Mathpower uniformly by 15%. We consider the single-scenario optimization over 15 years of the planning horizon. Performance of the single-scenario solver for this example is illustrated in Fig. 2, Fig. 3 and Fig. 4.

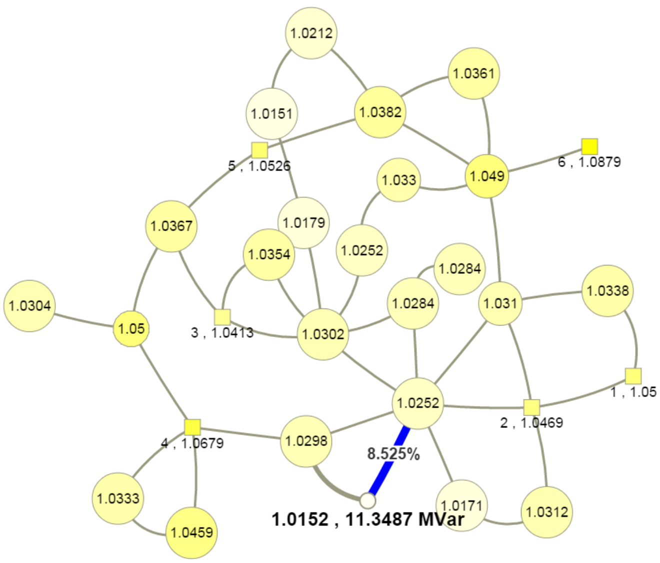

Fig. 3. Visualization of the solver output. Circles and squares mark consumers and generators. Voltage profile is shown in color (transitioning from yellow for maximum voltage to white for the minimal voltage). Line marked blue was initially overloaded and it was also selected by the solver for SC correction/placement. Number, shown next to the blue line, shows correction (in percentage). Node which was chosen for the (only) SVC correction is shown as a white dot. Bold numbers which appear next to the dot show level of the voltage and corrected/installed reactive power provided at the optimal solution.

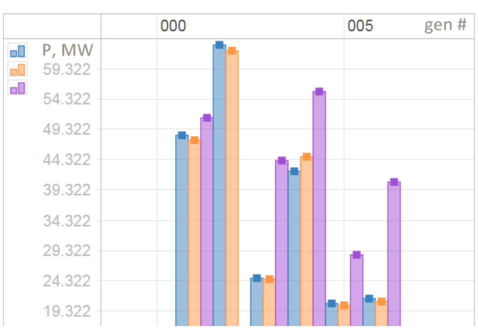

Fig. 4. Comparison of the active power dispatch (P, MW) at six generators of the 30-bus system for three different configurations marked as blue, orange and purple, where the color-coding is identical to the one used in Fig. (2) .

We observe that the optimal correction is achieved with investment in one SC device and one SVC device. Reactive powers are also corrected at the generators. Notice that in spite of the fact that the reactive power correction comes at no cost at the generators, the congestion forces the optimization to build an SVC device for an additional reactive correction far from generators.

Next we illustrate performance of our multiple-scenario solver with the example accounting for 10 configuration scenarios.

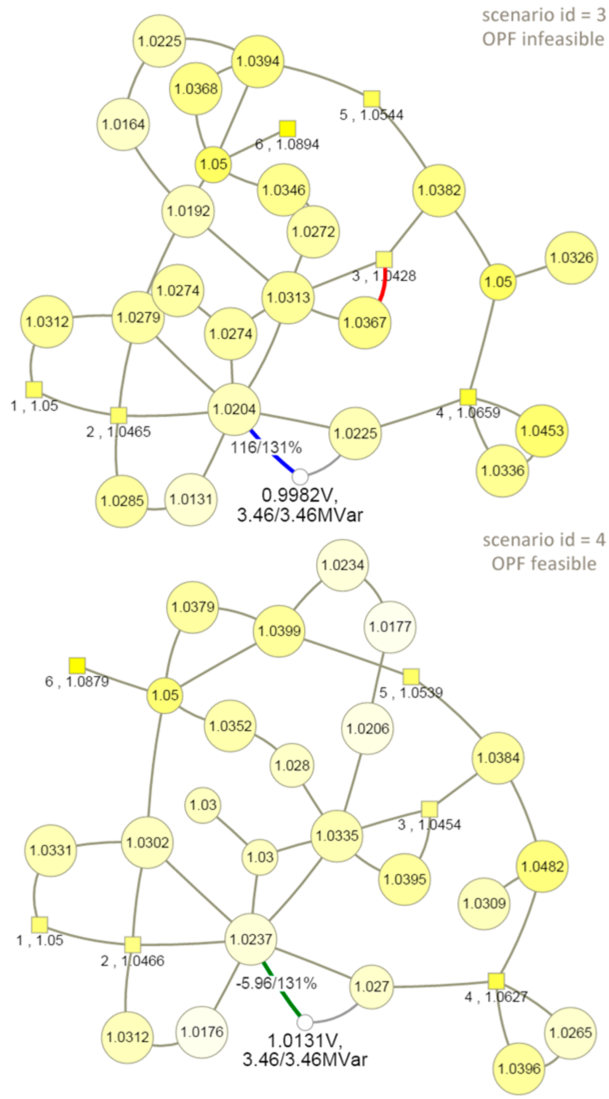

Fig. 5. Visualization of the final states for two scenarios (out of 10 considered). Marking and color-coding (yellow-to-white) is the same as given in the captions of Fig. 3. Additionally we show "actual setting of the device"/"capacity of the device".Initially overloaded and compensated line is marked blue, initially overloaded line which was relieved without placement of a SC devise on it is shown red, and line which is chosen for compensation even though it was not overloaded prior to the correction is shown green.

The scenarios are considered on equal footing, i.e. each occurring with the probability of 10%. Each of the ten configurations is generated overloading uniformly by 10% a feasible case of the Matpower with subsequent addition of random correction (sampled from the Gaussian zero mean distribution with standard deviation equal to the 10% of the load position at the node).

The system is optimized over the year-long time horizon. In this case some of the scenarios are originally OPF-feasible and others are not. Fig. 5 shows optimal voltage profiles and optimal settings for the installed devices over two (out of ten) exemplary scenarios. We observe that the multi-scenario algorithm discovers distinct feasible solutions for each of the scenario. The resulting optimal placement is sparse. Moreover, once a device is installed it is not utilized at its maximum in all the scenarios.

Progress of the algorithm is shown in Fig. 6. Left y-axis of Fig. 6 illustrates convergence of the total cost (value of the objective function). We find that the multi-scenario solver requires approximately 30 iterations to converge. Right y-axis portion of Fig. 6 illustrates gradual (and generally non-monotonic) reduction of line overloads.

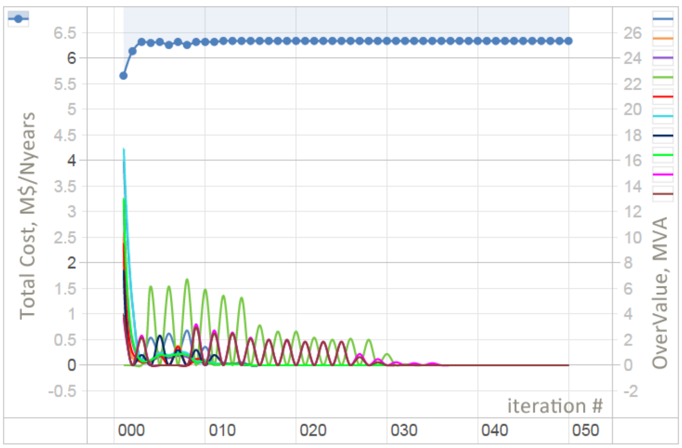

Fig. 6. Illustration of the algorithm dynamics: we show dependence of line overloads in MVA (lines numbered on the right) and dependence of the total cost in M$/years (blue dotted line shadowed from above) on the number of iterations.

VI. Conclusions and Future Works

In this manuscript we have developed new optimization framework for placement and sizing of FACTS devices in the power transmission systems. We worked within the exact AC PF paradigm and accounted for many properly weighted load configurations. Our problem formulation can be considered as generalizing standard AC OPF approach. The generalization consists of (a) modifying cost of the OPF to account, in addition to the standard cost of the generation dispatch, for the cost of FACTS installation (also promoting sparsity of solution); (b) allowing operational FACTS controls, different for individual loads but all within the limits of the state of installation. We have constructed efficient heuristics for solving this nonlinear and non-convex optimization. Our solver builds a convergent sequence of convex optimizations with linear constraints. Each constraint is represented explicitly through exact linearization of the originally nonlinear constraints (e.g. representing power flows and apparent power line limits) over all the degrees of freedoms (including FACTS corrections) around the current operational point. Performance of the solver is illustrated on our enabling example of the 30 node Matpower model.

Following extensions of this work are in our plans: (a) algorithm improvement, e.g. evaluating constraints only when needed (cutting plane), (b) demonstration of scalability (for thousands-node large systems), and (c) model generalizations, e.g. accounting for other installation and control options like these related to phase-shifters, line switching and also improving modeling of costs.

VII. Acknowledgment

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References

[1] N. Yamaguchi, I. Takahashi, H. Kita, K. Nishiya, and J. Hasegawa, "An allocation of wheeling costs in power systems with facts devices," in *Electric Utility Deregulation and Restructuring and Power Technologies, 2000. Proceedings. DRPT 2000. International Conference on*, 2000, pp. 549 –553.

[2] J.Zhou, Y. Yuan, and J. Yu, "Research on transmission congestion of power system containing wind farms based on genetic algorithm," in *Electric Utility Deregulation and Restructuring and Power Technologies (DRPT), 2011 4th International Conference on*, july 2011, pp. 104 –108.

[3] D. Waniek, U. Hager, C. Rehtanz, and E. Handschin, "Influences of wind energy on the operation of transmission systems," in *Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century, 2008 IEEE*, july 2008, pp. 1 –8.

[4] L. Fleischman, R. Cleetus, J. Deyette, S. Clemmer, and S. Frenkel, "Ripe for retirement: An economic analysis of the u.s. coal fleet," *The Electricity Journal*, vol. 26, pp. 51–63, 2013.

[5] M. Noroozian, L. Angquist, M. Ghandhari, and G. Andersson, "Use of upfc for optimal power flow control," *Power Delivery, IEEE Transactions on*, vol. 12, no. 4, pp. 1629 –1634, oct 1997.

[6] D. Gotham and G. Heydt, "Power flow control and power flow studies for systems with facts devices," *Power Systems, IEEE Transactions on*, vol. 13, no. 1, pp. 60 –65, feb 1998.

[7] G. Glanzmann and G. Andersson, "Using facts devices to resolve congestions in transmission grids," in *CIGRE/IEEE PES, 2005. International Symposium*, oct. 2005, pp. 347 – 354. [Online]. Available: \url{http://www.eeh.ee.ethz.ch/uploads/tx_ethpublications/Glanzmann_CIGRE_05.pdf}

[8] C. Rehtanz and U. Hager, "Coordinated wide area control of facts for congestion management," in *Electric Utility Deregulation and Restructuring and Power Technologies, 2008. DRPT 2008. Third International Conference on*, april 2008, pp. 130 –135.

[9] "Proposed terms and definitions for flexible ac transmission system (facts)," *Power Delivery, IEEE Transactions on*, vol. 12, no. 4, pp. 1848 –1853, oct 1997.

[10] N. Hingorani and L. Gyugyi, *Understanding FACTS concepts and technology of flexible AC transmission systems*. IEEE Press, New York, 2000.

[11] R. Mathur and R. Varma, *Thyristor-based FACTS controllers for electrical transmission systems*. IEEE Press, Piscataway, 2002.

[12] "Flexible ac transmission systems (facts)," http://en.wikipedia.org/wiki/Flexible_AC_transmission_system.

[13] V. Frolov, S. Backhaus, and M. Chertkov, "Efficient algorithm for locating and sizing series compensation devices in large power transmission grids: I. model implementation," *New Journal of Physics*, vol. 16, no. 10, p. 105015, 2014.

[14] ——, "Efficient algorithm for locating and sizing series compensation devices in large power transmission grids: Ii. solutions and applications," *New Journal of Physics*, vol. 16, no. 10, p. 105016, 2014.

[15] "Gurobi optimizaer, http://www.gurobi.com/."

[16] "Ibm cplex optimizer, http://www-01.ibm.com/software/commerce/optimization/cplex-optimizer/."

[17] R. D. Zimmerman and C. E. Murillo-Sanchez, "A matlab power system simulation package, http://www.pserc.cornell.edu//matpower/."