Approximation algorithms for some generalizations of TSP*

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The Traveling Salesman Problem (TSP) is the classic combinatorial optimization problem introduced in 1959 by G.Dantzig and J.Ramser in their seminal paper concerning an optimal scheduling for a fleet of gasoline trucks to service a network of gas-stations. It is curious that the authors believed that the problem they stated is tractable and can be efficiently solved to optimality for any instance. Now, thanks to R.Karp and C.Papadimitriu, we know that TSP is strongly NP-hard and remains intractable even in the Euclidean plane. Therefore, due to the well-known $P \neq NP$ conjecture, efficient optimal algorithms for TSP will hardly be designed ever. Furthermore, it is known that, in its general setting, TCP can not be approximated efficiently with any reasonable accuracy, since it has no $O(2^n)$-ratio polynomial time approximation algorithms unless $P = NP$. Meanwhile, in more specific (but acceptable for numerous applications) settings, there are known many promising approximation results, among them are famous 3/2-approximation N.Christofides algorithm for the metric TSP and S.Arora's Polynomial Time Approximation Schemes (PTAS) for fixed dimensional Euclidean spaces.

Recently it was proven that some known and valuable for applications generalizations of TSP have the similar complexity status and approximability behavior. In this talk we consider the k-Size Cycle Cover Problem, where a given edge-weighted complete (di)graph should be covered by k vertex-disjoint cycles of minimum total weight. For $k = 1$, this problem coincides with TSP and intractable. On the other hand, for unbound k, the problem is equivalent to the minimum weight perfect matching problem and can be solved to optimal in polynomial time. We show that, for any fixed k, k-SCCP is strongly NP-hard even in the plane, inapproximable in general setting, belongs to APX for any metric and has EPTAS in d-dimensional Euclidean space for any fixed $d > 1$.

The second topic of this lecture is concerned with another important generalization of the TSP known as Generalized Traveling Salesman Problem or GTSP. Along with edge-weighted graph $G(V, E, w)$, the instance of GTSP is defined by partition $V = V_1 \cup \ldots \cup V_k$ of its nodeset. The goal is to find a minimum weight cyclic tour visiting each cluster V_i at one node. Recently, it was shown thatGTSP can be solved to optimality efficiently in the class of quasi-pyramidal tours, which generalizes the well known pyramidal tours for the classic TSP. Further, for geometric settings of the problem several PTASs were proposed.

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