On the abnormality in open shop scheduling

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We consider the classic open shop scheduling problem to minimize the makespan [1]. An input I of the problem can be described by an matrix of processing time $P = (p_{ji})_{m\times n}$, m and n being the numbers of machines and jobs respectively. The *standard lower bound* for instance I is defined as $\bar{C}(I) \doteq \max\left\{\max\limits_i \sum\limits_j p_{ji}, \max\limits_j \sum\limits_i p_{ji}\right\}$. Let us denote *the total load* of instance I by $\Delta(I) \doteq \sum\limits_{i,j} p_{ji}$. Note that by definition $\Delta(I) \leqslant m\bar{C}(I)$.

A feasible schedule S for instance I is referred to as *normal* if its makespan $C_{\max}(S)$ coincides with $\bar{C}(I)$ [2]. An instance I is *normal* if a normal schedule for I exists. It is well known that any two-machine open shop instance is normal [1] while for $m \geqslant 3$ that is not the case.

For any instance I we define its *abnormality* as $\alpha(I) \doteq C^*_{\max}(I)/\bar{C}(I)$, where $C^*_{\max}(I)$ is the makespan of optimal schedule for I. The natural question is, how large can an abnormality of some instance be.

It was shown in [3] that the maximal abnormality for any three-machine open shop instance if equal to $\frac{4}{3}$. That value is achieved on the instance I' with the following matrix of processing times $\left(\begin{smallmatrix}1&2&0&0\\1&0&2&0\\1&0&0&2\end{smallmatrix}\right)$. Note that the total load of I' reaches an extremal value of $3\bar{C}(I')$.

In this paper we discuss the maximal abnormality for the three-machine open shop as a function of the total load (specifying our previous result from [3]). More precisely, let $\mathcal{I}_m(x) \doteq \{I \text{ is an instance of } m\text{-machine open shop} | \Delta(I) \leqslant x\bar{C}(I)\}$. Then we consider the following *abnormality function* $F_m(x) \doteq \sup\limits_{I\in\mathcal{I}_m(x)} \alpha(I)$. We show that $\forall m \leqslant 2\, \forall x \in [1,2]\ F_m(x) = 1$ and $\forall x \geqslant 2\ F_m(x) \leqslant x/2$, and describe "almost exact" form of function $F_3(x)$.

References

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