Weber Problem on Line with Forbidden Gaps

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The Weber problem on a line with forbidden gaps is defined as follows. It is necessary to locate facilities $X_1, \ldots, X_n$, which are the segments, on a straight–line segment of length LS containing some fixed segments (forbidden gaps) $F_1, \ldots, F_m$. Location in forbidden gaps is not allowed. Denote the centers of X_i and F_j by x_i and b_j respectively; the lengths of X_i and F_j by l_i and p_j respectively; $w_{ij} \geq 0$, $u_{ik} \geq 0$ — the specific costs of connections between X_i and F_j, X_i and X_k. The target is to locate $X_1, \ldots, X_n$ on the line outside $F_1, \ldots, F_m$ and so that they do not intersect with each other and the total cost of the connections between facilities among themselves and facilities with gaps is minimized. The mathematical model of the problem is [1]:

$$G(x_1, \ldots, x_n) = \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij}|x_i - b_j| + \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} u_{ik}|x_i - x_k| \to \min, \quad (1)$$

$$|x_i - b_j| \geq \frac{l_i + p_j}{2}, \quad i = 1, \ldots, n, \quad j = 1, \ldots, m, \quad (2)$$

$$|x_i - x_k| \geq \frac{l_i + l_k}{2}, \quad i, k = 1, \ldots, n, \quad i < k, \quad (3)$$

$$\frac{l_i}{2} \leq x_i \leq LS - \frac{l_i}{2}, \quad i = 1, \ldots, n. \quad (4)$$

The problem (1)–(4) is NP-hard and it is reduced to series discrete subproblems [1]. In [2] a definition of local optimum of the problem and variants of lower bounds of goal function of the subproblems are proposed.

In this paper a branch and bounds algorithm for solving of the problem is developed. A computational experiment with use of this algorithm and the approximate algorithm from [1] is made.

References

1. Zabudskii, G.,G., Veremchuk, N.,S.: An Algorithm for Finding an Approximate Solution to the Weber Problem on a Line with Forbidden Gaps. Diskret. Anal. Issled. Oper. 23(1), 82-96 (2016) [J. Appl. Ind. Math. 10(1), 136–144 (2016) DOI: 10.1134/S1990478916010154]
2. Gennady Zabudsky, Natalia Veremchuk: About Local Optimum of the Weber Problem on Line with Forbidden Gaps. Proc. DOOR 2016, Vladivostok, Russia, September 19–23, 2016. CEUR-WS. 2016. Vol. 1623. P. 115–124. CEUR-WS.org, online http://ceur-ws.org/Vol-1623/paperco17.pdf

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