A polynomial 3/5-approximation algorithm for the problem of finding three edge disjoint Hamiltonian circuits of the maximum weight in a complete digraph

Toktokhoeva Surena*

Novosibirsk State University, 2 Pirogova Str., Novosibirsk 630090, Russia

The m-Peripatetic Salesman Problem (m-PSP) (introduced by Krarup in 1975) is a natural generalization of the Traveling Salesman Problem (TSP). In m-PSP one need to find m edge disjoint Hamiltonian cycles of minimum or maximum total weight in a complete weighted graph. In this paper we investigate the asymmetric maximization version of the m-Peripatetic Salesman Problem (m-APSP-max). The input of the problem is a complete directed graph G and a non-negative weight function of its edges. The task is to find m edge disjoint Hamiltonian circuits of maximum total weight in G.

It is known that the problems of finding one or more Hamiltonian circuits in a digraph are NP-hard. So the efforts of most researchers are concentrated on finding cases where the problem can be solved in polynomial time and developing polynomial algorithms with guaranteed approximation ratios for such problems.

The best known approximation algorithm for the ATSP-max has the guaranteed ratio 2/3 [1]. Authors of [2] developed an algorithm with the same ratio for the 2-APSP-max and an algorithm on random instances for the m-APSP for which the conditions of its asymptotical exactness were established.

However, for $m > 2$, no deterministic approximation algorithms for the m-APSP-max are constructed. The main result of this paper is a deterministic Algorithm for the 3-APSP-max with approximation ratio 3/5 and cubic running time. Following the ideas in [2], our Algorithm starts with an acyclic edge colouring of a specified regular subgraph in G. Such a colouring allows us to construct three partial tours with sufficiently large weight in G and to extend them to three edge disjoint Hamiltonian circuits which form a desired 3/5-approximate solution of the 3-APSP-max.

References

1. Kaplan, H., Lewenstein, M., Shafrir, N., Sviridenko, M.: Approximation algorithms for asymmetric TSP by decomposing directed regular multigraphs. JACM. V.52(4), 602–626 (2005)
2. Gimadi, E.Kh., Glebov, A.N., Skretneva, A.A., Tsidulko, O.Yu., Zambalaeva, D.Zh.: Combinatorial algorithms with performance guarantees for finding several Hamiltonian circuits in a complete directed weighted graph. Discrete Applied Mathematics. V.196(11). 54–61 (2015)

* The research is supported by RFBR (project 16-01-00108)