Algorithms of Inertial Mirror Descent in Convex Optimization Problems

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Consider a problem of minimizing the mathematical expectation of a convex loss function on a given convex compact set of a finite-dimensional real space E with a norm $\|\cdot\|$. The oracle produces unbiased stochastic subgradients of the loss function at current points with a uniformly bounded second moment of dual norm in E^*. The goal is to modify the well-known method of Mirror Descent (MD) proposed in 1979 by A.S. Nemirovskii and D.B. Yudin [1] and generalized the famous gradient method from the Euclidean case to an arbitrary primal-dual pair of spaces (E, E^*); see also [2] and the references therein. In this paper (cf. [3]):

- The idea of a new, so-called inertial MD method is demonstrated on the example of a deterministic optimization problem with continuous time; in particular, in the Euclidean case the heavy ball method [4] is realized; it is noted that the new method does not use additional averaging;
- The discrete algorithm of the inertial MD is described; the theorem on the upper bound on the regret is proved (i.e., the difference between the current mean loss value and the minimum value) for the problem of stochastic optimization;
- An illustrative computational example is given.

In the conclusion, we discuss future work on both deterministic and stochastic algorithms of inertial mirror descent.

References

1. Nemirovskii, A. S. and Yudin, D. B., *Slozhnost' zadach i effektivnost' metodov optimizatsii*, Moscow: Nauka, 1979 (in Russian). Translated under the title *Problem Complexity and Method Efficiency in Optimization*, Chichester: Wiley, 1983.
2. Juditsky, A. B., Nazin, A. V., Tsybakov, A. B., and N. Vayatis, N.: Recursive aggregation of estimators by the mirror descent algorithm with averaging // Problems of Information Transmission, 41(4):368–384, 2005. Translated from Problemy Peredachi Informatsii, No.4, 2005, pp.78–96.
3. Nazin, A. V.: Algorithms of Inertial Mirror Descent in Convex Problems of Stochastic Optimization // Automation and Remote Control. 2017. Submitted.
4. Polyak, B. T.: Some methods of speeding up the convergence of iteration methods // USSR Computational Mathematics and Mathematical Physics, 4(5):1–17, 1964.

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