

Solution of bilevel optimization problems using the KKT approach

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The optimistic version of the bilevel optimization problem reads as

$$\min_{x,y} \{F(x, y) : G(x) \leq 0, (x, y) \in \mathbf{gph}\Psi\}, \quad (0.1)$$

where $\mathbf{gph}\Psi$ is the solution set mapping of the so-called lower level problem

$$\Psi(x) = \underset{y}{\text{Argmin}} \{f(x, y) : g(x, y) \leq 0\}. \quad (0.2)$$

All functions $F, f, g_i : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}$, $i = 1, \dots, p$ and $G : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^q$ are assumed to be smooth, the functions $y \mapsto f(x, y)$ and $y \mapsto g_i(x, y)$, $i = 1, \dots, p$ are convex. Problem (0.1), (0.2) has many applications in very different fields, see e.g. [1]. The problem is a nonconvex, nonsmooth optimization problem. This can be seen if we consider e.g. the linear bilevel optimization problem where the feasible set of (0.1) equals the union of faces of a polyhedron describing the feasible set of the lower level problem, see [3].

One method often used is to replace the lower level problem by its Karush-Kuhn-Tucker conditions provided some regularity condition is satisfied. This results in

$$\begin{aligned} F(x, y) &\rightarrow \min_{x,y,u}, \\ G(x) &\leq 0, \\ \nabla_y L(x, y, u) &= 0, \\ g(x, y) &\leq 0, \\ u &\geq 0, \\ u^\top g(x, y) &= 0, \end{aligned} \quad (0.3)$$

where $L(x, y, u) = f(x, y) + u^\top g(x, y)$ denotes the Lagrange function of (0.2). In [2] it is shown that both problems (0.1) and (0.3) are equivalent if global optimal solutions are computed. But, problem (0.3) is a nonconvex optimization problem for which the Mangasarian-Fromovitz constraint qualification is violated at every feasible point [5]. One promising approach for solving such problems uses a relaxation of the complementarity slackness conditions

of (0.3), see [6]. This approach can be shown to converge to certain stationary solution of (0.3). Unfortunately, stationary solutions of (0.3) are in general not related to stationary solutions of the bilevel problem (0.1), see [2]. Using one more approximation, Mersha [4] was able to show convergence to Bouligand stationary solutions of (0.1) under very restrictive assumptions.

In the talk we will show that local optimal solutions of the problem

$$\begin{aligned}
 F(x, y) &\rightarrow \min_{x, y, u} \\
 G(x) &\leq 0 \\
 \|\nabla_y L(x, y, u)\| &\leq \varepsilon_1 \\
 g(x, y) &\leq 0 \\
 u &\geq 0 \\
 -u^\top g(x, y) &\leq \varepsilon_2
 \end{aligned} \tag{0.4}$$

converge for $\varepsilon \downarrow 0$ to local optimal solutions of (0.1) under weak assumptions. Problem (0.4) can be solved using standard solution algorithms. Using variational analysis we will also show that stationary solutions of (0.4) converge to C -stationary solutions of (0.3) for $\varepsilon \downarrow 0$ provided the MPEC-MFCQ is satisfied for (0.3).

References

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