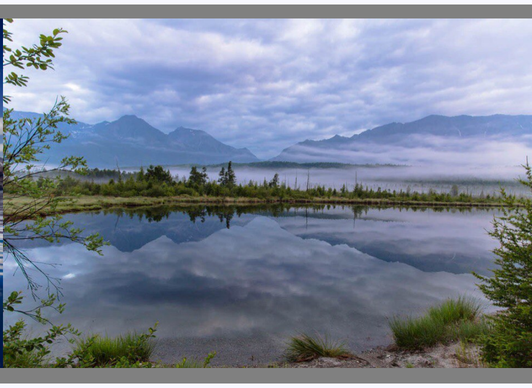
XVII Baikal International School-Seminar

METHODS OF OPTIMIZATION AND THEIR APPLICATIONS

Abstracts





July 31 – August 6, 2017
Maksimikha, Buryatia

Melentiev Energy Systems Institute SB RAS

Russian Foundation for Basic Research

Laboratory of Algorithms and Technologies for Networks Analysis,
Higher School of Economics, Nizhny Novgorod

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2017

UDC 519.6+519.7+519.8

Abstracts of the 17th Baikal international school-seminar “Methods of Optimization and Their Applications”. Irkutsk: ESI SB RAS, 2017, 224 p.

This volume contains abstracts of the talks devoted to theory and methods of linear, convex, nonlinear programming, discrete and global optimization, multicriteria optimization and game theory, as well as to software and software packages for solving various mathematical programming problems.

For researchers specializing in corresponding fields of applied mathematics.

The conference is organized with the financial support of

Russian Foundation for Basic Research (project no. 17-01-20340)

Laboratory of Algorithms and Technologies for Networks Analysis,
Higher School of Economics, Nizhny Novgorod

Melentiev Energy Systems Institute
of Siberian Branch of the Russian Academy of Sciences

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Институт систем энергетики им. Л. А. Мелентьева СО РАН

Российский фонд фундаментальных исследований

Лаборатория алгоритмов и технологий анализа сетевых структур национального исследовательского университета «Высшая школа экономики», Нижний Новгород

XVII Байкальская международная школа-семинар

МЕТОДЫ ОПТИМИЗАЦИИ И ИХ ПРИЛОЖЕНИЯ

Тезисы докладов

31 июля – 6 августа 2017 г.
с. Максимиха, Бурятия

Иркутск
2017

УДК 519.6+519.7+519.8

Тезисы докладов XVII Байкальской международной школы-семинара «Методы оптимизации и их приложения». Иркутск: ИСЭМ СО РАН. – 2017. – 224 с.

В данном томе представлены работы, посвященные теории и методам линейного, выпуклого, нелинейного программирования, дискретной и глобальной оптимизации, многокритериальной оптимизации и теории игр, а также программам и программным комплексам для решения различных задач математического программирования.

Для научных работников, студентов и аспирантов, специализирующихся в соответствующих областях прикладной математики.

Конференция организована при финансовой поддержке

Российского фонда фундаментальных исследований
(номер проекта 17-01-20340)

Лаборатории алгоритмов и технологий анализа сетевых структур
национального исследовательского университета
«Высшая школа экономики», Нижний Новгород

Института систем энергетики им. Л. А. Мелентьева
Сибирского отделения Российской академии наук

Ответственные за выпуск: *Колосницын А. В.*
Минарченко И. М.
Дресвянская Н. В.
д.ф.-м.н. Хамисов О. В.

Фотографии на обложке: Владимир Бурмейстер

CONTENTS

PLENARY TALKS

Saddle-point methods for solving terminal control problems with phase constraints

Anatoly Antipin

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We consider a problem of terminal control with phase constraints and the boundary value problem

$$x_1^*, x(t), u^*(t)) \in \text{Argmin}\{\langle \varphi_1(x_1)\rangle \mid G_1 x_1 \le g_1,\ x_1 \in X_1 \subseteq R^n, \tag{1}$$

$$\frac{d}{dt}x(t) = D(t)x(t) + B(t)u(t),\ t_0 \le t \le t_1, \tag{2}$$

$$x(t_0) = x_0 \in R^n, x^*(t_1) = x_1^* \in X_1 \subset R^n, \tag{3}$$

$$G(t)x(t) \le g(t),\ x(\cdot) \in AC^n[t_0, t_1],\ u(t) \in U\},$$

$$u(\cdot) \in U = \{u(\cdot) \in L_2^r[t_0, t_1] |\ \|u(\cdot)\|_{L_2} \le \text{const}\}. \tag{4}$$

Here $D(t), B(t) - n \times n, n \times r$ are matrix functions, continuously depending on time, G_1 $-m \times n$, $(m \le n)$ is a fixed matrix, g_1, x_0 are given vectors. The controls $u(\cdot)$ are elements of the space $L_2^r[t_0, t_1]$. U is a convex closed set. We introduce the linearized Lagrange function. Using the Lagrange function, we can formulate sufficient saddle-saddle conditions for the extremum for the problem of terminal control with phase constraints and the boundary value problem in the form convex programming.

$$\frac{d}{dt}x^*(t) = D(t)x^*(t) + B(t)u^*(t), \quad x^*(t_0) = x_0, \tag{1}$$

$$p_1^* = \pi_+(p_1^* + \alpha(G_1 x_1^* - g_1)), \tag{2}$$

$$\eta^*(t) = \pi_+(\eta^*(t) + \alpha(G(t)x^*(t) - g(t))), \tag{3}$$

$$\frac{d}{dt}\psi^*(t) + D^{\mathrm{T}}(t)\psi^*(t) + G^{\mathrm{T}}(t)\eta^*(t) = 0, \quad \psi_1^* = \nabla\varphi_1(x_1^*) + G_1^{\mathrm{T}} p_1^*, \tag{4}$$

$$u^*(t) = \pi_U(u^*(t) - \alpha B^{\mathrm{T}}(t)\psi^*(t)), \tag{5}$$

where $\pi_+(\cdot), \pi_+(\cdot), \pi_U(\cdot)$ – projetion operators, respectively, onto the positive orthant R_+^m, onto the positive orthant $\Psi_+^n[t_0, t_1]$, $\alpha > 0$, and onto the set of controls U.

Using sufficient conditions, iterative saddle-point methods can be formulated. These methods converge in all components of the solution, namely: convergence in controls is weak, convergence in phase and conjugate trajectories is strong (in the norm of space). Convergence in terminal variables is also strong.

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A reduction of cardinality to comlementarity in sparse optimization

Oleg Burdakov

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We present a reformulation of the cardinality constraint optimization problem (CardCP) as a mathematical program in continuous variables with complementarity-type constraints. The two problems are equivalent in the sense that they have the same global minimizers. A relation between their local minimizers is also discussed. Local optimality conditions for CardCPs are derived on the base of representing cardinality constraints in a disjunctive form. A continuous reformulation of portfolio optimization problem with semi-continuous variables and cardinality constraint is given. Results of numerical experiments are presented.

Joint work with Christian Kanzow (University of Würzburg, Germany) and Alexandra Schwartz (Technical University of Darmstadt, Germany).

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A Heuristic Algorithm for the Mixed Integer Setup Knapsack Problem

Altannar Chinchuluun

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We study the mixed integer setup knapsack problem (SKP). In the classical knapsack problem, we are given a set of items S each having a a profit and cost. We are also provided with a budget and the objective is to pick a set of items with maximum profit such that the cost of the set is within the budget. Items could be fractionally picked. For SKP, in addition to the above, we are provided with a partition of the set S and we incur a fixed cost in the objective on the partition for any item picked within the partition. We provide a greedy heuristics for the general case of the problem and provide some computational results to show the efficiency of the heuristics. For the special case, where we have exactly one item in a partition, we show that the problem has a polynomial time approximation scheme.

Solution of bilevel optimization problems using the KKT approach

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The optimistic version of the bilevel optimization problem reads as

$$\min_{x,y}\{F(x,y) : G(x) \leq 0,\ (x,y) \in \mathbf{gph}\Psi\}, \tag{1}$$

where $\mathbf{gph}\Psi$ is the solution set mapping of the so-called lower level problem

$$\Psi(x) = \underset{y}{\text{Argmin}}\ \{f(x,y) : g(x,y) \leq 0\}. \tag{2}$$

All functions $F, f, g_i : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R},\ i = 1,\dots,p$ and $G : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^q$ are assumed to be smooth, the functions $y \mapsto f(x,y)$ and $y \mapsto g_i(x,y),\ i = 1,\dots,p$ are convex. Problem (1), (2) has many applications in very different fields, see e.g. [1]. The problem is a nonconvex, nonsmooth optimization problem. This can be seen if we consider e.g. the linear bilevel optimization problem where the feasible set of (1) equals the union of faces of a polyhedron describing the feasible set of the lower level problem, see [3].

One method often used is to replace the lower level problem by its Karush-Kuhn-Tucker conditions provided some regularity condition is satisfied. This results in

$$\begin{array}{rcl} F(x,y) & \to & \min\limits_{x,y,u}, \\ G(x) & \leq & 0, \\ \nabla_y L(x,y,u) & = & 0, \\ g(x,y) & \leq & 0, \\ u & \geq & 0, \\ u^\top g(x,y) & = & 0, \end{array} \tag{3}$$

where $L(x,y,u) = f(x,y) + u^\top g(x,y)$ denotes the Lagrange function of (2). In [2] it is shown that both problems (1) and (3) are equivalent if global optimal solutions are computed. But, problem (3) is a nonconvex optimization problem for which the Mangasarian-Fromovitz constraint qualification is violated at every feasible point [5]. One promising approach for solving such problems uses a relaxation of the complementarity slackness conditions of (3), see [6]. This approach can be shown to converge to certain stationary solution of (3). Unfortunately, stationary solutions of (3) are in general not related to stationary solutions of the bilevel problem (1), see [2]. Using one more approximation, Mersha [4] was able to show convergence to Bouligand stationary solutions of (1) under very restrictive assumptions.

In the talk we will show that local optimal solutions of the problem

$$\begin{array}{rcl} F(x,y) & \to & \min\limits_{x,y,u} \\ G(x) & \leq & 0 \\ \|\nabla_y L(x,y,u)\| & \leq & \varepsilon_1 \\ g(x,y) & \leq & 0 \\ u & \geq & 0 \\ -u^\top g(x,y) & \leq & \varepsilon_2 \end{array} \tag{4}$$

converge for $\varepsilon \downarrow 0$ to local optimal solutions of (1) under weak assumptions. Problem (4) can be solved using standard solution algorithms. Using variational analysis we will also show that stationary solutions of (4) converge to C-stationary solutions of (3) for $\varepsilon \downarrow 0$ provided the MPEC-MFCQ is satisfied for (3).

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Measurability of Optimal Strategies in the Stochastic Optimal Control Problem with Discrete Time

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Abstract. The paper investigates the existence problem of optimal measurable strategies in the control problem of the stochastic system with discrete time and probability criterion.

Keywords: measurability, the probability criterion, optimal control, dynamic programming method, diversification.

The problem of optimal control of the stochastic system with discrete time and probability criterion is investigated. To solve the problem the dynamic programming method is applied.

Note that it is hard enough to find optimal strategies even for the criterion in the form of mathematical expectation and several steps in stochastic system, since there are numerous difficulties for calculation of Bellman's functions at the each step. That's why investigators focus on obtaining either analytical results or some approximation to optimal strategies. But formal application of the dynamic programming method can generates measureless strategies that imply to uncertainty in the probability functional or mathematical expectation.

At the present paper we state that the continuity of the transition function from a current state to the next state, the lower semi-continuity of the terminal state function, and the independence of random factors that arise at each step are required for existence of optimal measurable strategies in recurrence relations of the dynamic programming method.

An example of the problem of portfolio selection is considered. It's established that Bellman's functions are continuous except for one point.

Algorithms of Inertial Mirror Descent in Convex Optimization Problems

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Consider a problem of minimizing the mathematical expectation of a convex loss function on a given convex compact set of a finite-dimensional real space E with a norm $\|\cdot\|$. The oracle produces unbiased stochastic subgradients of the loss function at current points with a uniformly bounded second moment of dual norm in E^*. The goal is to modify the well-known method of Mirror Descent (MD) proposed in 1979 by A.S. Nemirovskii and D.B. Yudin [1] and generalized the famous gradient method from the Euclidean case to an arbitrary primal-dual pair of spaces (E, E^*); see also [2] and the references therein. In this paper (cf. [3]):

- The idea of a new, so-called inertial MD method is demonstrated on the example of a deterministic optimization problem with continuous time; in particular, in the Euclidean case the heavy ball method [4] is realized; it is noted that the new method does not use additional averaging;
- The discrete algorithm of the inertial MD is described; the theorem on the upper bound on the regret is proved (i.e., the difference between the current mean loss value and the minimum value) for the problem of stochastic optimization;
- An illustrative computational example is given.

In the conclusion, we discuss future work on both deterministic and stochastic algorithms of inertial mirror descent.

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Multivariate Algorithmics: On Interactions with Heuristics

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Parameterized and, more generally, multivariate algorithm design is mainly tailored towards identifying "tractable" special cases for "intractable" (that is, typically NP-hard) problems. Ideally, this leads to efficient algorithms providing optimal solutions. The central observation herein is that if some problem-specific parameters are small, then certain problems can be solved efficiently by confining exponential running time growth to the parameters only. In real-world scenarios, most computationally hard problems are attacked with heuristic approaches, that is, often simple (in particular, greedy) algorithms that are efficient but do not guarantee optimal solutions, or algorithms without provable running time guarantees.

A long-term goal of Theoretical Computer Science (analysis of algorithms and computational complexity) should be to contribute to a better understanding of the effectiveness of heuristics. In this talk, through some case studies including examples from graph-based data clustering, graph anonymization, matching in graphs, and computational social choice, we discuss some fruitful interactions between heuristics and parameterized algorithm design and analysis. We also discuss challenges for future research.

This talk is based on several results achieved during the last few years with various co-authors.

Integer Programming Approaches for Critical Elements Detection in Graphs

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Graph-theoretical models arise in a variety of application areas due to their elegance and inherent ability to logically represent (as edges) important relationships, e.g., communication nd transportation links, between structural elements (i.e., nodes) of complex systems. An interesting research question arising in this context is the problem of identifying the most "significant" nodes and edges of a given graph G, specifically, those whose removal (subject to a budgetary constraint) maximally degrades some structural properties G (e.g., connectivity, communication efficiency) according to some pre-defined metric. These nodes (edges) and the corresponding optimization problem are often referred to as critical nodes (edges) and the Critical Elements Detection problem, respectively. Most of the attention in the literature has been focused on the node version of the problem, also referred to as the Critical Node Detection (CNP) problem. The concept of critical nodes and edges allows for the characterization of vulnerability and robustness properties of a given networked system with respect to node and edge removals, which may be random failures or errors due to operating conditions, or natural disasters. In recent years, this stream of research has received a significant attention in the literature. In addition to apparent interpretations in telecommunication and transportation areas, the Critical Elements Detection problem has natural applications in a number of other important domains, e.g., social network analysis. For example, in social networks each node corresponds to a person, edges represent some type of relationships (or interactions) between the individuals (e.g., friendship, collaboration), and critical nodes are often referred to as the "key players" of the network (e.g., informal leaders of the organization or community). In this talk we review recent results on the topic of Critical Elements Detection in the Operations Research literature. Our focus will be on applications of integer programming approaches.

Nonlinear Programming Models of Power-Aware Scheduling and Cloud Computing

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In the off-line speed scaling problems of power-ware scheduling it is required to determine the processing speed of each job either on a single machine or on parallel machines. The speeds are selected in such a way that (i) the cost of speed changing, often understood as energy needed to maintain a certain speed, is minimized, and (ii) the actual processing time of each job allows its preemptive processing within a given time window. We propose a new methodology for the speed scaling problem based on its link to scheduling with controllable processing times and submodular optimization. We reduce the speed scaling problems to a generic problem of minimizing a convex separable function over a base polyhedron and adapt a decomposition algorithm by Fujishige for its solution. This results in faster algorithms for traditional speed scaling models, characterized by a common speed/energy function. In addition, it handles efficiently the most general models with job-dependent speed/energy functions, which to the best of our knowledge have not been addressed prior to this study. In particular, the running time of the improved algorithm that solves the general version of the single-machine problem depends quadratically on the number of tasks to be scheduled.

Additionally, we address speed scaling problems that arise in cloud computing, where it is becoming a standard service level to start computing a customer's task on its arrival, which is possible due to abundant resource of the cloud.

Energy markets: optimization of transport infrastructure

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Keywords: energy markets, transport infrastructure, social welfare.

Markets of natural gas, oil and electricity play an important role in economies of many countries. Every such market includes its own transmission system. Consumers and producers are located at different nodes, and transmission capacities of the lines between the local markets are limited. The share of transport costs in the final price of the resource is typically substancial, so the problem of transport system's optimization is of practical interest. Vasin, Dailova (2014) determine the optimal transmission capacity for a two-node market. The present study considers a general problem of social welfare optimization with account of production costs, consumers' utilities and costs of trasmission capasities' increments. The complexity of the problem concerns with substancial fixed costs related to expansions of transmitting lines. If the set of expanded lines were given, the problem would be convex and could be solved by standard methods. However, under a big number of lines the efficient search of the set requires special tools. In general the problem of transport system optimization is NP-hard (Guisewite, Pardalos, 1990). Vasin, Dolmatova (2016) determine conditions for submodularity or supermodularity of the social welfare function on the set of transmitting lines for some types of networks. These properties provide a possibility to apply the known efficient optimization methods (see Khachaturov, 1989). However, they typically do not hold. We introduce more general concepts of competitive and supplementary transmitting lines which permit to develop similar methods for determination of the optimal set of expanded lines. For tree-type markets we establish conditions such that every pair of lines is either competitive or supplementary.

[1]The research was supported by Russian Foundation for Basic Research (project No. 16-01-00353/16).

TUTORIAL LECTURES

Rate of convergence to the local minimum for the 0,1,2-order method in global optimization

Alexander Gasnikov

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We plane to describe recent results in global optimization of G. Lan, S. Ghadimi, E. Hazan, Yu. Nesterov, B. Polyak. Note, that all the results describe the rate of convergence in terms of the number of iterations.

We consider 0-order methods (at each iteration one can only calculate the value of the function), 1-order methods (at each iteration one can calculate the gradient of the function), 2-order methods (at each iteration one can calculate Hessian of the function). We show how these approaches can be adaptively tune in parameters, that characterized the smoothness of the function. We also investigate how sensitive are the methods of global optimization to unpredictable error of small level arises on iterations.

Parallel Computation for Time-Consuming Multicriterial Optimization Problems

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In the paper, an efficient method of solving the time-consuming multicriterial optimization problems, where the optimality criteria can be multiextremal and computing the criteria values can require a large amount of computations has been proposed. The proposed approach is based on the reduction of the multicriterial problems to the global optimization ones using the minimax convolution of the partial criteria, the dimensionality reduction with the use of the Peano space-filling curves, and the application of the efficient parallel information-statistical global optimization methods [1, 2]. The key aspect of the developed approach consists in the overcoming of the high computational complexity of the global search of the set of the efficient solutions in solving the multicriterial optimization problems. A considerable improvement of the efficiency and an essential reduction of the amount of computations have been provided by means of the maximal possible utilization of the whole search information obtained in the course of computations. Within the framework of the developed approach, the methods for reducing the whole available search information to the values of current scalar nonlinear programming problem being solved have been proposed. The search information is used by the optimization methods for the adaptive planning of the executed global search iter-ations.

The results of the numerical experiments have demonstrated such an approach to allow reducing the computation costs of solving the multicriterial optimization problems considerably – tens and hundreds times.

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[1]This work has been supported by Russian Science Foundation, project No 16-11-10150 ”Novel efficient methods and software tools for time-consuming decision making problems using superior-performance supercomputers.”

Approximation algorithms for some generalizations of TSP[1]

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The Traveling Salesman Problem (TSP) is the classic combinatorial optimization problem introduced in 1959 by G.Dantzig and J.Ramser in their seminal paper concerning an optimal scheduling for a fleet of gasoline trucks to service a network of gas-stations. It is curious that the authors believed that the problem they stated is tractable and can be efficiently solved to optimality for any instance. Now, thanks to R.Karp and C.Papadimitriu, we know that TSP is strongly NP-hard and remains intractable even in the Euclidean plane. Therefore, due to the well-known $P \neq NP$ conjecture, efficient optimal algorithms for TSP will hardly be designed ever. Furthermore, it is known that, in its general setting, TCP can not be approximated efficiently with any reasonable accuracy, since it has no $O(2^n)$-ratio polynomial time approximation algorithms unless $P = NP$. Meanwhile, in more specific (but acceptable for numerous applications) settings, there are known many promising approximation results, among them are famous 3/2-approximation N.Christofides algorithm for the metric TSP and S.Arora's Polynomial Time Approximation Schemes (PTAS) for fixed dimensional Euclidean spaces.

Recently it was proven that some known and valuable for applications generalizations of TSP have the similar complexity status and approximability behavior. In this talk we consider the k-Size Cycle Cover Problem, where a given edge-weighted complete (di)graph should be covered by k vertex-disjoint cycles of minimum total weight. For $k = 1$, this problem coincides with TSP and intractable. On the other hand, for unbound k, the problem is equivalent to the minimum weight perfect matching problem and can be solved to optimal in polynomial time. We show that, for any fixed k, k-SCCP is strongly NP-hard even in the plane, inapproximable in general setting, belongs to APX for any metric and has EPTAS in d-dimensional Euclidean space for any fixed $d > 1$.

The second topic of this lecture is concerned with another important generalization of the TSP known as Generalized Traveling Salesman Problem or GTSP. Along with edge-weighted graph $G(V, E, w)$, the instance of GTSP is defined by partition $V = V_1 \cup \ldots \cup V_k$ of its node-set. The goal is to find a minimum weight cyclic tour visiting each cluster V_i at one node. Recently, it was shown thatGTSP can be solved to optimality efficiently in the class of quasi-pyramidal tours, which generalizes the well known pyramidal tours for the classic TSP. Further, for geometric settings of the problem several PTASs were proposed.

[1] Supported by RSCF grant 14-11-00109

Mathematical heuristics for combinatorial optimization problems

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In the last decades metaheuristics have demonstrated the ability to solve hard combinatorial problems of practical sizes within reasonable computational time. Simulated Annealing, Tabu Search, Variable Neighborhood Search, GRASP, evolutionary – inspired algorithms like Genetic Algorithms, Ant Colony Optimization and several other paradigms have established their value in various application areas: facility location, vehicle routing, scheduling and others. In this tutorial we will discuss the main ideas of some metaheuristics and their hybrids, in particular, with the classical mathematical programming methods, so-called matheuristics. Specifically, we will describe the follower hybrid methods:

- Local Branching
- Core Concepts
- Feasible Pump
- Relaxation Induced Neighborhood Search (RINS)
- Large Neighborhoods and some others.

Application of the methods for NP-hard problems in combinatorial optimization and bi-level programming will be discussed as well.

Networking Games

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Nowadays the fields of science related with artificial intelligence, information resource allocation, optimization of computer resources are rapidly increasing. It was appeared new terminology – telecommunication mathematics, networking games and others. In many network problems, one would observe the dominance of transportation and telecommunication interpretations.

The theme of the course is devoted to discussion of new results in this direction, mostly, game-theoretic methods in networks. The most important here are the problems of competitive routing, transportation games, behavioral equilibrium, analysis of social networks, comparison of selfish and cooperative behavior and the problems of appropriate management.

Hub location problem – models and methods

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Hubs are special facilities used in real-world transportation networks. The role of hub nodes in hub networks is to collect, consolidate, transfer and distribute flow. The transportation between non-hub nodes in hub networks is not accomplished directly, but via hub nodes to whom non-hub nodes are assigned. The main advantage of hub networks is that the cost required for their establishment is less than the cost needed for establishing a network in which any two nodes are connected directly. One of the earliest applications of hub location problems involve the design and management of telecommunication and transportation systems, such as computer and satellite networks, logistical systems, airline industry, postal delivery systems, cargo transportation, etc. Nowadays, there are many other domains that may use the concept of hub networks, such as: freight transportation, fast delivery systems, public transit, maritime industry, and many others.

In this tutorial talk, I will first present possible classification of hub location problems and then introduce some new more realistic models. For all hub location problem types listed, a mathematical programming models will be shown. In the second part of the tutorial, I will talk about heuristic solution methods recently published or submitted, that were proposed by me and my collaborators.

Sharp Penalty Mapping Approach to Approximate Solution of Variational Inequalities and Discrete-time Lyapunov Theory for Optimization Processes

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Part 1. Sharp Penalty Mapping Approach to Approximate Solution of Variational Inequalities

We consider here a generalization of penalty functions approach to solution of variational inequalities, which is based on replacing a potential mapping, associated with the gradient field of a penalty function with the oriented field of a sharp penalty mapping. It gives an additional freedom in construction of iteration schemes for approximate solution with controllable accuracy of variational inequalities of high dimensionality, for instance for solving transportation equilibrium problems.

Part 2. Discrete-time Lyapunov Theory for Optimization Processes

This part is devoted to some new tools for studying convergence properties of iteration processes common in optimization and related areas. Despite proliferation and successes of a great number of heuristics in machine learning, automatic classification, discrete optimization and other subjects, there is an everlasting need for full verification of validity of such processes, which not only guarantees their correct application, but quite often shows the ways for the improvements. As a rule such analysis amounts to proofs of convergence of sequences of approximate solutions to the desirable exact solution and is based on Lyapunov-like statements about relaxation properties of a related sequences of values of convergence indicators. There are several general nontrivial methods for establishing such properties, but new problems in Big Data areas ask for a new approaches oriented in particular on algorithms for obtaining approximate solutions of huge-scale optimization and equilibrium problems. Some new ideas in area are discussed and demonstrated in this lecture.

Interval Methods for Data Fitting under Inaccuracy and Uncertainty

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How to estimate parameters from measurements subject to errors and uncertainty?

The measurement errors are often supposed to be random quantities that can be adequately described by the probability theory. Such data fitting problems are the subject of consideration in classical regression analysis. In particular, when we know that the measurement errors are normally distributed with zero mean, then the Maximum Likelihood Method leads to the popular least squares estimates.

In many situations, however, we do not know the shape of the error distribution and its parameters. It may even happen that the probability theory cannot be adequately applied to the data fitting problem, since the sample is too small and/or statistical stability is violated. Instead, we only know that the measurement errors are located on a certain interval. Then, for the solution of the data fitting problem, we can exploit new approaches based on the methods of interval analysis and adjacent disciplines.

In our lectures, we give an overview of the subject, beginning with the pioneering work by Leonid Kantorovich [1] up to the latest most significant results in this area. We analyze specificity and drawbacks of the interval approaches to data fitting under essential interval uncertainty in data and discuss their implementation, especially in the part related to numerical optimization.

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CONTINUOUS OPTIMIZATION

Application of the Fast Automatic Differentiation to the Computation of the Gradient of the Energy of Atoms' System

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For modeling solid atomic structures the Tersoff Potential is often used. The Tersoff Potential depends on ten parameters specific to the modeling material. These parameters are usually unknown and they should be identified as the solution of the inverse problem. In [1] one possible optimization problem was considered. It consists of minimizing the following cost function

$$f(\xi) = \sum_{i=1}^{m} \omega_i (y_i(\xi) - \widetilde{y}_i)^2 \tag{1}$$

where ω_i is the weight factor; $\widetilde{y}_i$ is the value of the i-th material characteristic obtained experimentally, and $y_i(\xi)$ is the value of the same material characteristic calculated using Tersoff Potential with ξ parameters ($\xi \in R^m$ are vector parameters to be identified). The solution of the problem is looked for on the set $X \subseteq R^m$, which is a parallelepiped. A required set of parameters has to provide the minimum deviation of the calculated characteristics of the material from the known experimental values. For numerical solution of this problem the gradient minimization methods are often used. One of the terms in formula (1) is the total energy of the system of atoms. There exists the need to calculate efficiently the exact gradient of the total energy with respect to parameters of the Tersoff Potential.

This gradient is often calculated (see, for example, [1]) using the finite difference method. Studies have shown that finite difference method does not allow to calculate the gradient of the energy of atoms' system with respect to Tersoff parameters with acceptable accuracy and requires times to calculate the value of the function.

It should be noted that the above-mentioned optimization problem is solved with a determened, fixed position of atoms of the considered basic crystal structure. Solving the problem of parameters identifying in such a statemant, there is no certainty that the positions of basic atoms will correspond to the minimum of potential energy of the system. Therefore, the following step of studies is the optimization relatively coordinates of particles, which arranges particles to the positions, which correspond to the minimum of summary potential energy of the considered system of atoms. At this stage, there is a need to determine the gradient of the energy of atoms' system with respect to the coordinates of the atoms.

In this work, we build a multistep algorithm to calculate the value of total atoms' system energy in the case where this energy is determined by Tersoff Potential and a multistep algorithm to calculate the conjugate variables, by which the value of the above-mentioned gradients are determined with machine precision on the basis of the Fast Automatic Differentiation methodology.

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Numerical Study of Sparse Optimization Methods for PageRank Problem

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Keywords: huge-scale optimization, sparsity, Frank–Wolfe conditional gradient method, coordinate descent method, direct gradient method in L1–norm.

PageRank vector search is one of the most famous modern huge-scale problem. Initially, it is the task of finding of eigenvector corresponding to eigenvalue 1 of the (column) stochastic matrix: $Px = x$, where P is web-graph sparse matrix which element P_{ij} is non-zero only if page j (graph node) cites page i. The original PageRank problem can be reduced to the optimization form in different ways, in the paper we discuss the following version:

$$f(x) = \frac{1}{2}\|Ax\|_2^2 \to \min_{\langle x,e\rangle=1}, \tag{1}$$

where $A = P^T - I$, I is identity matrix, $e = (1, \dots, 1)^T$, $x \in \mathbb{R}^n_+$.

The most interesting and important feature of PageRank problem is the sparsity of solution. For “close-to-reality” web-graphs (i.e. non-synthetic cases) we can find almost sparse vector x which will provide solution with required accuracy. This great feature allows us to use sparse optimization methods which based on ideas of Yuri Nesterov [1]. Such methods provides ideology of “lightweight iterations” which modifies only 1-2 variables of x and then performs *update* of function value instead it’s traditional full re-calculation. Such update technique explicitly uses problem’s internal structure and allows significantly reduce the cost of iteration [2].

The paper presents results of numerical study of problem (1) with number of sparse optimization methods: direct gradient method in L1–norm, Frank–Wolfe conditional gradient method, deterministic and randomized variants of coordinate descent methods. The methods behavior and properties was studied with different simulated and real data sets. The results of numerical experiments confirmed the efficiency of proposed approaches.

Acknowledgments. The research was partly supported by the RFBR, project no. 16-07-00664.

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Makespan Lower Bound For Resource-Constrained Project Scheduling Problem With Time-Dependent Resource Capacities

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Keywords: Project Scheduling, RCPSP, Makespan Lower Bound.

Resource Constrained Project Scheduling Problem (RCPSP) is a well-known NP-hard scheduling theory problem. Recent surveys [1], [2] showed that existing estimation approaches for makespan lower bounds are based on polynomial and exponential algorithms. Polynomial algorithms are not able to provide good bounds for instances with complex precedence relations graphs. Exponential algorithms have very high computational complexity for large-scaled instances. This research focused on the development of a new makespan lower bound estimation algorithm able to find a good lower bound in reasonable time. For the formulation with time-dependent resource capacities pseudo-polynomial algorithm with the complexity $O(n^3r(n+m+r)H\log H)$ operations is presented, where n – number of jobs, r – number of resources, $H = \sum_{j\in N} p_j$ – makespan upper bound, m – number of breakpoints of resource capacity functions in time horizon H. Numerical experiments using well-known PSPLIB benchmark [3] and real data instances show that the suggested algorithm is useful especially for large-scaled instances. For 5 PSPLIB instances algorithm outperforms existed approaches and improves lower bound.

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Multi-Clouds Workload Distribution for the Secure and Reliable Storage of Data under Uncertainty

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Designing IT infrastructures based on cloud technologies must take into account the emerging risks of the security, and reliability due to numerous types of uncertainties associated with cloud computing [Tch1, Cha1].

In this paper, we study systems for storing and processing data in the clouds with the use of residual number systems under conditions of uncertainty. These systems can be represented as oriented graphs $M_{i,j}$, and the workload can be grouped by cloud providers that have different processing power, reliability, and data security.

The basic idea of the load distribution between cloud providers is to increase the probability that each of the cloud systems performs processing at equal intervals T. If any of the cloud providers can not complete the calculations with an acceptable probability, then the computing load is redistributed among other providers or additional computing capacity has to be purchased from this provider.

As an objective function, the sum of the product p_i (probability of data processing V_i by the i-cloud provider in time t) and P_i (probability of access to the results of data processing by an i-cloud provider). It is necessary to determine the values of $M_{i,t}$ for which the objective function is maximal:

$$\sum_{k=1}^{n} \left(p_i\left(M_{i,t}\right) \cdot P_i\left(M_{i,t}\right)\right) \to \max$$

under conditions of:

1. Security of data storage
2. Security of data processing
3. Project budget
4. Data encoding time in the RNS
5. The decoding time in the Binary.

To solve this problem, we have developed software in the C# programming language in the Microsoft Visual Studio 2015 programming environment, which allows to simulate a distributed data storage and processing system with real cloud providers: DropBox, Box, Google, Mail, YandexDisk and others.

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Generalized Concavity and Global Optimization

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We suggest an approach to solve special classes of multi-extremal problems—optimization the monotone combination (e.g., sum, product) of several functions, under the that there are known the effective algorithms to optimize each of this item (e.g., each of these functions has some properties of generalized concavity.) The algorithm proposed is iterative. It realizes the idea of the branch-and-bound method, consists in successive correcting the bounds of optimal value of objective functions. Moreover we use the methodology of multi-objective optimization, studying the image of Pareto boundary in the image space. In each iteration, the total area of the region, guaranteed to contain the image optimal point, decreases at least twice. We discuss the applicability to marketing models: optimization of communication expenditure [5] and the effectiveness of advertising [6], pricing [3]; to monopolistic competition models: retailing [2], investments in R&D [1], market distortion [3], and international trade [2].

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Application of Bioinspired Optimization Artificial Intelligence Technology for Solution Task of Cryptanalysis of Encryption Systems

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The report is devoted to a problem of use of new technologies of artificial intelligence — the bioinspired optimizing methods and algorithms imitating processes of evolution of wildlife for the solution of a task of cryptanalysis of classical and modern systems of enciphering. Application of genetic algorithms for cryptanalysis of classical codes of shifts, replacements along with experimental results is considered [1]. Also the possibility of cryptanalysis of the XOR codes for a some of special cases is investigated (a rejection of statistical characteristics from a random stream, reuse of parts of scale, modification of data in the channel, aprioristic information about the form and structure of the document). In addition, the possibility of the solution of a task of cryptanalysis with use of new models of the bioinspired methods — algorithms of ant colonies and a bee swarm is described [2]. Distinctive features of these methods are considered, and also use of the bioinspired technologies for cryptanalysis of classical codes of shifts and replacement, asymmetric cryptosystems based on decision-theoretic task of cryptography (factorization of numbers and finding factors of a number), and modern block encryption standards along with some experimental results (the DES, AES, Russia standard, on the basis of determination of quantity of optimum symbols of the deciphered text at the cryptanalysis of type 2) is investigated [3]. In addition, the actual problem of development of the combined bioinspired technologies combining the main features of classical "natural" algorithms is considered. The combined bioinspired methods including the main operations of genetic algorithms, and also algorithms of ant and bee colonies are presented, demonstration examples of their implementation are provided.

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Minimization of polyhedral function over hypercube using projections

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We consider a rather classical problem of nonsmooth optimization — minimization of polyhedral function with interval constraints on variables, i.e.

$$\min_{\mathbf{B}} \max_{i=1,...,N} a_i^T x + c_i, \quad \mathbf{B} = \{x \in \mathbb{R}^n : x_{imin} \leq x_i \leq x_{imax}, i = 1, ..., n\}.$$

It is clear that one can apply linear programming or some kind of subgradient algorithm to solve this problem. Despite that, we investigate a new approach based on conversion of the problem into new setting — finding an intersection between a special polytope (*zonotope*) and a line [1].

Zonotope is an affine transformation of m - dimensional cube:

$$\mathbf{Z} = \{z \in \mathbb{R}^n : z = z_0 + Hw, \ ||w||_\infty \leq 1\}, \, w \in \mathbb{R}^m, \, n \leq m.$$

First, we transform our problem into the form

$$\gamma \to \min, \quad Ax + c + y = \bar{1}\gamma,$$

where $\bar{1}$ is a vector of ones and y is a vector of auxiliary variables (also box-constrained). Then our problem is reduced to the following:

$$\gamma \to \min, \quad \bar{1}\gamma \in \mathbf{Z}, \quad w = [x^T \ y^T]^T.$$

If we know an interior point of $\mathbf{Z}$ on the line $\bar{1}\gamma$, it is possible to derive a linearly convergent algorithm based on bisection of interval on the line. At each iteration (the number of iterations can be computed in advance for the given accuracy) we apply an algorithm (e.g. Frank-Wolfe [2] or Nesterov fast gradient) to find a projection of a point on the line onto the zonotope (which is equivalent to projection onto a hypercube). Then (if the current point is outside $\mathbf{Z}$) we take a point of intersection between the line and the achieved level set of distance function as our next point. The algorithm can be used as e.g. a part of d.c. optimization in the style of [3].

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Projection on ellipsoid with $O(1/t^2)$ convergence rate

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We consider a simple problem of finding a point on the boundary of n-dimensional ellipsoid, which is the closest to some given point x_g outside the ellipsoid with respect to some differentiable distance function $f(x)$, i.e.

$$\min f(x - x_g), \quad x \in S = \{x : (x - x_0)^T Q(x - x_0) \leq 1\},$$

where Q is a positive definite symmetric matrix. This problem statement occurs in some applications, such as obstacle avoidance in robotics or trust region approach in smooth optimization [1].

In recent paper [2] it was shown that for strongly convex sets, the vanilla Frank-Wolfe optimization algorithm gives a rate $O(1/t^2)$, where t is the number of iterations. We simply apply the FW algorithm to our problem, after that the main subproblem we consider is how to compute minimum of a linear function $\nabla f^T x$ on our ellipsoid, where ∇f is the function gradient at the current point.

This task is easy to achieve with the Cholesky decomposition [3] of matrix $Q = L^T L$. We introduce a linear transformation $z = Lx$, after which our ellipsoid corresponds to a (hyper)sphere, while new linear objective function is given by $\nabla f^T L^{-1} z$. The computation of its minimum on $Z = \{z : (z - z_0)^T (z - z_0) \leq 1\}$ is then straightforward and given by a closed formula.

We do hope that due to its simplicity the algorithm can find its application in some real-time settings.The author is obliged to G.Sh. Tamasyan (Saint-Petersburg State University) for problem statement.

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A new branch and bound algorithm for the bilevel linear programming problem

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Abstract. The bilevel linear programming problem is considered. A new branch and bound algorithm in combination with optimization technique based on the support functions is proposed. Computational results for randomly generated test problems are given and analyzed.
Keywords: bilevel optimization, support functions, branch and bound algorithm

The statement of the bilevel linear programming problem (BLPP) in which the feasible region of the upper-level problem is determined implicitly by the solution set of the lower-level problem is investigated. The objective function and the constraints of the upper-level and lower-level problems of BLPP are all linear and affine. However, even for this simplest two-level linear case the problem is strongly NP-hard.

For this class of problems, the branch and bound algorithm is the most successful algorithm to deal with the complementary constraints arising from Karush-Kuhn-Tucker conditions.

In this paper we propose a novel implementation of the branch and bound algorithm based on the support functions. It is assumed that in each point of the convex set containing the feasible region, the functions that define the feasible set have distinct support functions. The method is used to expedite the solution finding process.

The feasibility and effectiveness of the proposed approach are demonstrated and the test results are compared with those of other methods reported in the literature.

Acknowledgements. The research was supported by grant 15-07-08986 from the Russian Foundation for Basic Research.

Universal Intermediate Gradient Method with Inexact Oracle

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In this paper, we consider first-order methods for convex optimization. The recent renaissance of these methods was mostly motivated by large-scale problems in data analysis, imaging, machine learning. In 2013 Nesterov proposed a Universal Fast Gradient Method (UFGM) which can solve a broad class of convex problems with Hölder-continuous subgradient and is uniformly optimal over this class. In 2013, Devolder, Glineur and Nesterov proposed an Intermediate Gradient Method (IGM) for problems with inexact oracle, which interpolates between GM and FGM to exploit the trade-off between the rate of convergence and the rate of error accumulation.

In this paper, we present Universal Intermediate Gradient Method (UIGM) for problems with deterministic inexact oracle. Our method both enjoys the universality with respect to Hölder class of the problem and interpolates between Universal Gradient Method and UFGM, thus, allowing to balance the rate of convergence of the method and rate of the error accumulation. First, we consider a composite convex optimization problem $\min_{x\in Q}\left[F(x) = f(x) + h(x)\right]$ where Q is a closed convex set, h is a simple convex function and function f is convex and subdifferentiable on Q with inexact Hölder-continuous subgradient. We assume that problem is solvable with optimal solution x^*. For such problems we construct our method and prove the theorem on its convergence rate. If the error δ of the oracle satisfies $\mathrm{O}\left(\frac{\varepsilon}{N^{p-1}}\right)$, where N is the number of algorithm steps, our method generates a point y s.t. $F(y) - F_* \leq \varepsilon$ in $\mathrm{O}\left(\inf\limits_{\nu\in[0,1]}\left(\frac{L_\nu R^{1+\nu}}{\varepsilon}\right)^{\frac{2}{1+2p\nu-\nu}}\right)$ interations, where $\nu \in [0,1]$ is the Hölder parameter, L_ν is the Hölder constant, $p \in [0,1]$ is the method parameter, R is an estimate for the distance from the starting point to the solution.

Then, under additional assumption, that F is strongly convex function with known constant μ, we use restart technique to obtain an algorithm with faster rate of convergence. We restart when $\frac{4}{\mu A_k}\omega_n \leq 1$. In our method A_k and ω_n is easy computed parameteres. If the error δ of the oracle satisfies $\mathrm{O}\left(\frac{\varepsilon}{N^{p-1}}\right)$, our method generates a point y s.t. $F(y) - F_* \leq \varepsilon$ in

$$N = \mathrm{O}\left(\inf_{\nu\in[0,1]}\left(\frac{L_\nu^{\frac{2}{1+\nu}}}{\mu\varepsilon^{\frac{1-\nu}{1+\nu}}}\omega_n\right)^{\frac{1+\nu}{1+2p\nu-\nu}}\cdot\left\lceil\ln\left(\frac{\mu R^2}{\varepsilon}\right)\right\rceil\right)$$

[3]This research was supported by a grant of the RF President MK-1806.2017.9.

D.c. programming approach to Malfatti's problem[1]

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We consider Malfatti's problem formulated 200 years ago. In 1803 Italian mathematician Malfatti posed the following problem: how to pack three non-overlapping circles of maximum total area in given triangle? In the beginning, Malfatti's problem was supposed to be solved in a geometric construction way. Malfatti originally assumed that the solution to this problem are three circles inscribed in a triangle such that each circle tangent to other two and touches two sides of the triangle. Now it is well known that Malfatti's solution is not optimal. The most common methods used for finding the best solutions to Malfatti's problem were algebraic and geometric approaches. In 1994 Zalgaller and Los [1] showed that the greedy arrangement is the best one. There is still a conjecture about solving Malfatti's problem for more than four circles by the greedy algorithm.

In [2,3] the problem has been formulated as the convex maximization problem over a nonconvex set and global optimality conditions by Strekalovsky [4] have been applied to this problem. In this talk, we formulate Malfatti's problem as D.C. programming problem with a nonconvex feasible set. For solving numerically Malfatti's problem, we apply an algorithm [5], which converges locally. For a computational purpose, we consider some test problems. Computational results are provided.

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[1]This research is supported by Russian Science Foundation, project No. 15-11-20015

The Global Optimization Approach to Robot's Workspace Determination

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The workspace is the set of points the end effector or the tool can reach. The topic has been investigated thoroughly for decades, and an interested reader should be referred to [1], where its authors suggest the following classification of the workspace assessment techniques: geometrical, algebraic, and discretization. The majority of these techniques are intended to be used only for robots within the class they were developed for. For instance, the geometric techniques are quite efficient, though they can be applied to relatively simple robots. The techniques that based on discretization can be applied to a wider class of parallel robots, though they time consuming and are intended to be used in cases when the forward or the inverse kinematic problems have a simple solution.

In this work, we present a general approach suitable for any robot whose workspace can be defined as a system of inequalities. This approach is based on the non-uniform covering technique [2] that we use to solve a system of non-linear inequalities. We illustrate the efficiency of the proposed approach using a real-life example of a planar parallel robot [3].

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Convex optimization in Hilbert space with applications to ill-posed problems

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We propose rather general approaches (based on the gradient type methods) to solve convex optimization problems in Hilbert space [2]. We are interested in the case when Hilbert space has infinite dimension. It doesn't typically allow to calculate the exact value of the gradient (Frechet derivative). So that we have a tradeoff between the cost of one iteration and the number of required iterations: one can calculate gradient exactly, so the convergence is fast (we may use fast gradient descent), but the cost of one iteration is large, vice versa, one can calculate the gradient roughly so the convergence is slow (we can use only robust the simple gradient method), but the cost of the one iteration is cheap. This investigation is motivated by the class of ill-posed problems for elliptic initial-boundary value problems (the Cauchy problem for the Laplace equation [1]).

Acknowledgments. The work was supported by the RFBR (grants 15-01-09230, 16-01-00755, 16-29-15120 and 17-51-540004), the Ministry of Education and Science of the Russian Federation.

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Global Search in Fractional Programming via D.C. Constraints Problem

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This paper addresses a rather general fractional optimization problem, i.e. the problem of optimizing the sum of several rational functions.

We reduce it to a problem with d.c. constraints [1] and develop the global search method based on the global optimality conditions for a problem with nonconvex (d.c.) constraints [2].

The global search method comprises two principal stages:
1) a local search [1], which provides an approximately critical point;
2) procedures of escaping from critical points (provided by a local search method).
This procedures consist in constructing an approximation of the level surface of the convex function which generates the basic nonconvexity in the problem, and solving the auxiliary linearized (at the points from the approximation) problems.

The global search algorithm for solving the sum-of-ratios fractional programming problem was verified on a set of low-dimensional test problems taken from literature as well as on randomly generated problems with up to 200 variables and 200 terms in the sum.

Acknowledgments. This work has been supported by the Russian Science Foundation, Project No. 15-11-20015.

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DC Programming Biobjective Approach for Solving an Applied Rougher Flotation Optimization Problem

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We propose a deterministic mathematical programming approach to optimizing the metallurgical performance and determining the best operating conditions of the rougher flotation process. Our methodology is based on several advanced modern mathematical programming and multiobjetive optimization techniques including DC programming and the special global search strategy [1], an exact penalty method [2], and the ε-constraint algorithm [3]. The aim is to find the operating conditions of the flotation process that lead to the concentrate grade and recovery maximization subject to some technological constraints.

We develop a framework for finding an approximation to the Pareto optimal set of the bi-objective DC problem that assumes its decomposition into series of subproblems (DC programs with a DC constraint). Each subproblem is then solved to global optimality, providing a Pareto optimal solution to the bi-objective DC problem. To this end, we develop a solution method based on the global search theory proposed in [1]. According to the theory based on the global optimality conditions, the process of searching for global optimal solutions to the nonconvex optimization subproblems consists of the two principal components: (i) a special local search procedure taking into account the structure of the problem, and (ii) a procedure of escaping from critical points found by the local search that presupposes using special global optimality conditions.

We demonstrate the effectiveness of the proposed approach by carrying out a case study for real rougher flotation of copper-molybdenum ores performed in the Erdenet Mining Corporation concentration plant (Erdenet, Mongolia).

Acknowledgements. This work has been supported by the Russian Science Foundation, Project No. 15-11-20015.

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Bilevel stochastic linear programming problem with quantile criterion and continuous random parameters

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Abstract. The bilevel stochastic programming problem with quantile criterion and continuous random parameters is considered. The lower level problem is linear. Properties of the bilevel problem are researched. To solve the problem, the confidence method is used.

Keywords: bilevel problem, stochastic programming, quantile criterion.

The statement of the bilevel stochastic programming problem with quantile criterion and continuous discribution of random parameters is suggested. In this problem, there are two decision makers: the leader and the follower.

The follower selects his strategy solving the so-called lower level optimization problem. He knows the realization of the random parameters and the leader's decision. This problem is assumed to be linear in the follower's strategy. The coefficients of the linear follower's objective function depend on the leader's strategy and the realization of the random parameters.

The leader chooses his strategy solving the upper level optimization problem. This problem is called the bilevel problem. In this work, the leader solves a stochastic minimization problem with quantile criterion. The leader knows only the distribution of the random parameters. The quantile criterion provides the minimal leader's loss that cannon be exceeded with a given probability. The leader's problem contains a constraint on the optimality of the follower's strategy in the lower level problem. The leader takes into account the follower's strategy as a function of the leader's strategy and the random parameters.

Conditions guaranteeing that the lower level problem has a unique solution with probability one are presented. This means that so-called optimistic and pessimistic solutions to the bilevel problem coincide.

To solve the problem, the confidence method is used. It is shown that the follower's optimal strategy is a discrete random vector. The confidence method allows us choosing realizations of this random vector that ensure the given level of probability. Then an optimal or suboptimal solution to the bilevel problem is selected.

Acknowledgements. The research was supported by the Russian Science Foundation grant 15-11-10009.

Decomposition Approach to Nonconvex Quadratic Programming[1]

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Consider a quadratic programming problem with nonconvex objective function and linear constraints, and reduce it by means of linear transformation to the form

$$\left.\begin{aligned} &\sum_{i=1}^{p} \lambda_i x_i^2 + \sum_{i=1}^{q} \mu_i y_i^2 + x^\intercal l_x + y^\intercal l_y \to \min_{(x,y)} , \\ &Ax + By \leqslant d, \qquad \underline{x} \leqslant x \leqslant \overline{x}, \qquad \underline{y} \leqslant y \leqslant \overline{y} , \end{aligned}\right\} \tag{1}$$

where $(\lambda, \mu) = (\lambda_1, \ldots, \lambda_p, \mu_1, \ldots, \mu_q)$ stand for eigenvalues of a matrix in the objective function, besides $\lambda < 0$, $\mu \geqslant 0$. For fixed x, consider the subproblem

$$\sum_{i=1}^{q} \mu_i y_i^2 + y^\intercal l_y \to \min_{y}, \qquad Ax + By \leqslant d, \qquad \underline{y} \leqslant y \leqslant \overline{y} , \tag{2}$$

that is, obviously, convex. Let $\varphi(x)$ be the optimal value function of (2). It is easy to determine that $\varphi(x)$ is convex function of x. Thus the problem (1) can be represented as

$$\sum_{i=1}^{p} \lambda_i x_i^2 + \varphi(x) + x^\intercal l_x \to \min_{x}, \qquad \underline{x} \leqslant x \leqslant \overline{x} . \tag{3}$$

Constructing iterative process in x, procedures for solving (3) can be obtained. For example, d.c. structure of the problem may be used. Decomposition described above is more effective when the dimension of y is significantly larger than the one of x, since the subproblem (2) causes no difficulties for present-day solvers.

[1]This work was supported by the Russian Foundation for Basic Research, project No. 15-07-08986.

Numerical study of bioinspired methods for solving global optimization problems

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Keywords: global optimization, bioinspired optimization methods, test problem, meta-heuristic approach.

Bioinspirated optimization methods performs models representing simplified analogues of some biological processes (population evolution, collective behavior of self-organizing agents, etc.). They refer to the so-called meta-heuristic approaches, which do not guarantee the global extrema finding, but often allows one to find a good approximation in a rather short time. In fact, meta-heuristics describe a set of rules for the search process implementation (mostly a stochastic process). It aims to find nearвЂ“optimal solution by the target function. In recent practice, large-scale problems with an "expansive" target functions are widely solved with the use of hybrids and multimethod schemes based on both bioinspired meta-heuristics and deterministic, in particular, gradient algorithms. Thus it becomes important to have some information about different meta-heuristics effectiveness and any preliminary assessment of their capabilities [1, 2, 3]. The work deals with the construction of an empirical rating, we perform a numerical study of more than 20 different modern bioinspired methods. For results of the research to be representative, we have defined a unified set of tests (synthetic and applied problems, dimensions of 100 or more variables). For each problems we generated a set of 100 starting points, each studied method runs on these sets with the same limit of an objective function computations. The main result of the work is a simple statistics (average, standard deviation, minimum and maximum value of the function, real time) of each method work on 100 starts for the whole set of the tests. Every considered method possessing its own parameters was searched for their possible values (using fixed step on a uniform grid). Thus, the results obtained are close to optimal in the sense of the methods parameters sets.

Acknowledgments. This work is partly supported by the Russian Foundation for Basic Research (project N 17-07-00627).

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Modeling of fairness for the kidney exchange problems with multiple agents[1]

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Kidney Exchange Programs (KEP) run in several countries and represent an alternative of transplant for patients in need of a kidney, with a willing donor that is physiologically incompatible with he/she, to still be transplanted. The underling optimization problem aims at maximizing the number of exchanges between such incompatible pairs. The problem can be represented on a directed graph and an exchange is a cycle in a graph. Nowadays it is common practice to include into KEP altruistic donors, persons willing to donate one of their kidneys, with no associated patient. Such donors may initiate a chain of exchanges, where the donor from the last incompatible pair donates to the first compatible patient in a waiting list. Bounds K and L are normally imposed on the length of cycle and chain, respectively, due to practical limitations. The kidney exchange problem is to find a set of vertex disjoint cycles and chains each of length at most K and L, respectively, that maximizes the total number of pairs in exchanges.

In a multi-agent framework we have a set of such programs that intent to collaborate jointly in order to increase total number of transplants performed. When maximizing the total number of transplants for the multi-agent pool, there may exist multiple optimal solutions, and some may benefit one agent more than others. It is essential to develop a procedure that ensures the selection of an optimal solution that will benefit equally all the agents in a long-term run. In this work, we considered different policies for pool management. To model the problem we used Integer Programming and proposed different models for fair distribution of transplants among agents. The models were validated and compared through exhaustive computational experiments.

[1]This work is financed by the ERDF – European Regional Development Fund through the Operational Programme for Competitiveness and Internationalisation – COMPETE 2020 Programme, and by National Funds through the Portuguese funding agency, FCT – Fundação para a Ciência e a Tecnologia, within project "mKEP - Models and optimisation algorithms for multicountry kidney exchange programs" (POCI-01-0145-FEDER-016677) and FCT project SFRH/BPD/101134/2014.

Approximation algorithms for intersecting straight line segments with equal disks[1]

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In this work approximation algorithms for the following problem are given.
Intersecting Plane Graph with Disks (IPGD): *given a simple plane graph $G = (V, E)$ and a constant $r > 0$, find the smallest cardinality set $C \subset \mathbb{Q}^2$ of points (disk centers) such that each edge $e \in E$ (which is a straight line segment) is within (Euclidean) distance r from some point $c = c(e) \in C$ or, equivalently, the disk of radius r centered at c intersects e.*
It can be reduced to the classical geometric Hitting Set problem with the set of objects $\{\{x \in \mathbb{R}^2 : d(x, e) \leq r\} : e \in E\}$ and the ground point set equal to $\mathbb{R}^2$, where $d(x, e)$ denotes Euclidean distance between a point $x \in \mathbb{R}^2$ and a segment $e \in E$. When segments of E have zero lengths, IPGD coincides with the known Disk Cover problem. Designing algorithms for it finds its applications in network security analysis [1]. The IPGD problem is intractable even in its simple cases.

Theorem 1. IPGD *is NP-complete even if segments from E do not intersect.*

Analogous hardness results are proved for special graphs G [2] of practical interest. Using approaches of [3] and [4], we give

Theorem 2. *A $400e$-approximate algorithm exists for* IPGD *which can be implemented in $O(|E|^4 \log^2 |E|)$ time and $O(|E|^2)$ space.*

We improve on the approach of [3] applied directly to IPGD both in complexity and approximation factor by using new ideas and data structures. Sharper approximation algorithms are also given for special graphs G of practical interest.

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[1]This work was supported by Russian Science Foundation, project №14-11-00109.

Computational Study of Conforming Behavior Phenomena in Random Networks

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For an arbitrary collective, comprising of a set of agents, the conforming behavior corresponds to that, when the agent makes decisions regarding its actions largely based on the opinions or actions of agents around it. Conformity as a social phenomenon can often be observed in real world. One of the first formal models of conforming behavior was proposed by M. Granovetter in [2]. In that paper it was assumed that collective were represented by complete graphs, thus each agent observed actions of all the other agents. The dynamics in [2] was defined via the number of active agents in the collective at particular time moments. In [2] there was considered the problem of finding stationary points — situations when the number of active agents at the next time moment coincides with that at the current time moment.

In [4] we described automaton models of conforming behavior. To represent collectives in [4] we used Synchronous Boolean Networks (SBN) [3]. From our point of view, the models from [4] make it possible to naturally interpret activation dynamics in networks defined by graphs of arbitrary structure. It was also shown that the proposed models possess relatively efficient algorithmic component. In computational experiments the state-of-the-art SAT solving algorithms [1] made it possible to analyze networks of random structure with up to 500 vertices.

In this report we present new computational results for models from [4]. In particular, we propose new techniques for constructing propositional encodings of automaton mappings, that make it possible to study activation dynamics (of models from [4]) for networks of thousands of vertices. In our computational experiments we considered networks generated according to Erdős-Rényi, Watts-Strogatz and Barabási-Albert models.

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Identification of Inactive Constraints in Convex Optimization Problems[1]

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We consider the problem of convex optimization in the next form

$$\begin{aligned} &f_0(x) \to \min, \\ &f_i(x) \leq 0,\ x \in \mathbb{R}^n, \end{aligned} \tag{1}$$

where $f_i(x)$, $i = 1, ..., m$ – convex not necessarily differentiable functions. We apply the simplex embeddings method [1]-[3] to solve the problem (1). This method is the analog of the well known ellipsoid method [4], [5]. In simplex embeddings method it is used $n-$dimensional simplexes instead of ellipsoids to localize the solution of a problem.

Consider the main idea of the method. Suppose that we have the start simplex S_0 on the step $k = 0$. This simplex contains the feasible set of the problem (1). We find the center $x^{c,0}$ of the simplex S_0 and construct the cutting hyperplane $L = \{x : g^T (x - x^{c,0}) = 0\}$ through the center, where $g \in \mathbb{R}^n$ is the subgradient of the objective function. Then we move to the next step $k = k + 1$ and immerse the simplex part that contains the solution to the problem into the new simplex S_1 which has the minimal volume. Using of this procedure let us construct simplexes that have less volumes than previous ones. Such algorithm provides the localization of the problem solution consistently. We stop the method when the simplex volume becomes quite small.

An exeption of inactive constraints is the unique feature of the simplex embeddings method. Suppose that we have a simplex which contains the solution of the problem (1). Denote by v^j the j-th simplex vertex, where $v \in \mathbb{R}^n$, $j = 1, ..., n + 1$. Also we have the list of the constraints $f_i(x) \leq 0$, $i = 1, ..., m$ from the problem (1). We must calculate the value of each function $f_i(x)$, $i = 1, ..., m$ in each simplex vertex to identify inactive constraints. Then we check further condition:

$$f_i\left(v^j\right) < 0, \quad i = 1, ..., m, \quad j = 1, ..., n + 1. \tag{2}$$

The i-th constraint from (2) is inactive if the inequality (2) is correct for each simplex vertex.

We apply the technique of inactive constraints identification on the set of convex programming problems. The results of numerical experiments are given in this paper.

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Constraint Programming for Cosmonauts Training Problem

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Abstract. A method based on constraint programming is proposed for solving a cosmonauts training problem. Computational results of the implemented method and experiments on real data are presented.
Keywords: operations research, scheduling models, constraint programming

We consider the following cosmonauts training problem (CTP). Each cosmonaut has his own set of tasks which should be performed with respect to resource and time constraints. The problem is to determine start moments for all considered tasks. CTP is a generalization of the resource-constrained project scheduling problem with "time windows". In addition, the investigated problem is extended with restrictions of the following sort. Let us assume that the set of tasks is split into several subsets. It is required to generate a schedule that the operations of one of the subsets are executed at least (or at most) with defined frequency. For details of CTP mathematical model, see Bronnikov et al. ([1]). Previously, for solving this problem the authors proposed an approach based on methods of integer linear programming (see Musatova et al. ([2])). However, this approach turned out to be ineffective for high-dimensional problems. A new method based on constraint programming is developed. A comparison of the two approaches is presented.

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Generalized Convexity with Operations Max and Min and its Applications in Economics

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We consider continuous nonnegative functions in space R^n_{++} which consists of the origin and of n-dimensional vectors with strictly positive components. We define idempotent/tropical analogues of the superlinear and sublinear functions and find their representations as optimums of functions which are idempotent/tropical analogues of the inner product.

Symbol $\wedge$ denotes a minimum of numbers or a component-wise minimum of vectors, and symbol $\vee$ denotes a maximum. If $\wedge$ is used instead of $+$ in the definition of subadditive function, one comes to the following definition. A function P is called min-subadditive if

$$P(x \wedge y) \leq P(x) \wedge P(y) \forall x, y \in R^n_{++}.$$

In a similar way, with $\vee$ and $\geq$, a max-superadditive function is defined.

Theorem 1. *The following statements are equivalent.*

(1) Function H increases. (2) Function H is min-subadditive. (3) Function H is max-superadditive.

Functions $(l, x) = \min_i l_i x_i$ and $[l, x] = \max_i l_i x_i$ serve as idempotent/tropical analogues of the inner product.

Theorem 2. *If H is an increasing first-degree-positively-homogeneous function, then there exists a set Λ such that*

$$H(x) = \max_{I \in \Lambda} (l, x) = \min_{l \in \Lambda} [l, x], x \in R^n_{++}.$$

We show that the set (the 'menu') Λ, as well as the functions (l, x), $[l, x]$ and the representations provided by Theorem 2, are useful in studying properties of basic economic objects, such as production functions, utility functions and corresponding models of production, economic growth, consumer behavior, happiness, etc.

Computational projection project

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Projection problems or the equivalent least norm problems $\min \|x\|_N, x \in X$ are widely used in theoretical studies as well as in computational practice with different norms $\| \cdot \|_N$ and different types of feasible sets X. In this communication apart from common polyhedral sets and polytopes the mixed orthogonal projection problems, projection on cones and their convolutions, dynamic decomposition methods for projection problems, difficulties of gaining high accuracy in projection problems and other topics will be considered.

The algorithmic approaches considered here form the basis of the Open Source project, based on ResearchGate [1] which is intended to stimulate theoretical and experimental studies of projection problems and develop a corresponding software.

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Comprehensive approach to optimization of network resources in a virtual data center

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Today, telecommunication networks are the basis for deploying various types of applications and services in data centers. The growing use of the cloud computing concept to provide access to applications and services every year increases the amount of converged network traffic. Physical network infrastructure of data centers do not always have time to opportunely adapt and scale existing solutions for the current tasks of the users. The main problem for data centers is a dynamically changeable structure of the circulating flow of network traffic. To improve the efficiency of using and scaling the existing architectures in the data centers, they use solutions based on the virtualization of resources. One of the complex solutions is the creation by using a software-defined infrastructure the virtual data centers. This solution is based on a software-defined network. But, existing approaches based on software defined-network don't provide enough flexible solutions, able to adapt to changes in traffic flows in real time. At peak load, this leads to an overabundance of traffic to specific physical network nodes which are not prepared to handle a large data flow.

For the efficient use of physical network resources necessary to carry out the monitoring and analysis of circulating traffic. This problem can be solved by using software and hardware solutions for monitoring objects and network resources of the data center.

In this paper are describes the development of an efficient algorithms for developed approaches for effective control of traffic flow in the virtual data center, based on the methods used in the data mining and machine learning to more accurately classify and identify flows of cloud applications and using network services for optimization of launch and deployment of applications and services in the virtual data center infrastructure using different placement methods. We propose an efficient algorithm for placing applications and services in the infrastructure of a virtual data center. The problem of optimization of placing service-oriented cloud applications by using the templates of virtual machines (VM) or containers with disabilities infrastructure virtual data center is reduced to the problem of packing in containers We also generalize the well renown heuristic and deterministic algorithms of Karmakar-Karp and Kor. We have developed an efficient algorithm to placing VMs by neural network optimization.

Our research has shown that static placement of containers on the physical nodes is not effective because it does not allow to redistribute the load quickly. Placing applications based on virtual machines due to the flexibility of load balancing showed better results, but the load on the compute nodes has increased considerably due to the additional overhead associated with the use of VMs. The most effective placement of the study was the use of containers inside the virtual machines. It is possible to increase the density of application hosting and control services and applications in the virtual data center, as well as allowed to place containers and data services and network applications in close proximity to each other thereby reducing the response time of applications on users queries. Thus increase the efficiency of the system.

On Some Computing Problems of Piecewise-Linear Function Approximations Solving by Linear Programming Tools

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In this report some computing problems connected with the piecewise-linear approximations of a given continuous function $f(x) : R^n \to R$ on the base of its values on a given set of the points are discussed. Such problems are often arising in the procedures of economical evaluation of the finance health of the different kinds of enterprises, in the numerical construction of the discriminate functions in pattern recognition applications, and in many other spheres [1,2]. They may be overburden very often by additional requirements playing a role of regularity conditions in the processes of the construction of such function approximations.

The function approximation search may be subjected to some quality merits as well as to some concrete requirements to the structure of the points that determinate the value $f(x)$. For example, such requirements often appear when the values of the function are computed with taking into account a partition of the points according to Delaunay's principle.

In this way authors have succeeded in the eliminating some defects and in generalization of some results from [3,4]. In particular, the approach discussed needs effective strategy of seeking such general linear programming problem solutions that belong to their facets of minimal dimension. In the report some technology of overcoming the specified difficulties is offered. It is partially based on an advanced linear programming tools (see, e. g. [5]).

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The algorithm of stochastic sampling in the investment program formation of the mega project

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Keywords: Network stochastic model, algorithm of stochastic sampling, megaproject, investment program

The report proposes an algorithm of stochastic sampling for the construction of the execution schedule of the network stochastic model and the formation of the investment program of the mega project. The task of implementing the megaproject investment program is presented as the task of optimizing resource-scheduling for alternative options for performing work with different probabilities. The algorithm of stochastic sampling of an individual realization of the network stochastic model consists in a sequential consideration of the groupings of work and the calculation of the temporal characteristics of events. As a result of stochastic sampling, some of the work is realized, the rest of the work is excluded from the list. The stochastic sampling of each work is determined by a specific description of the logical capabilities of the initial event and the final event of this work. Exception of work from the current implementation is done according to rule A, and inclusion of it in a particular implementation - according to rule B [1]. To reduce computational costs, branching is used. As a result, we obtain a family of deterministic network models. In each concrete implementation of the deterministic network model, the problem of finding the permissible schedule of the minimum duration at which the guidelines are fulfilled is solved. The total intensity of consumption of resources does not exceed their number. The problem of finding an acceptable schedule of the minimum duration is solved, at which the directive terms are fulfilled. The total intensity of consumption of resources does not exceed their number. The proposed algorithm is implemented on the real economic information of the megaproject of the East Siberian oil and gas complex. The schedule for the imple-mentation of the megaproject investment program and the distribution histogram of resources are constructed.

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Norm variability in Newton method for underdetermined systems of equations

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Newton method may serve as a tool for solution of underdetermined systems of algebraic (differentiable) equations $P(x) = 0,\ P : \mathbb{R}^n \to \mathbb{R}^m,\ m < n$. It is usually written via pseudo-inverse matrix, which correspond to Euclidean norms in pre-image and image spaces [1, 4]:

$$x^{k+1} = x^k - \alpha(P'(x^k))^{\dagger} P(x^k),$$

The same method can be used to explore image set of a non-linear differentiable mapping $\{g(x) : x \in \mathbb{R}^n\} \subseteq \mathbb{R}^m$, resulting in equations of type $g(x) = \gamma y$, with chosen direction y.

We propose variable-norm setup for Newton method as

$$z_k = \arg \min_{P'(x^k)z = P(x^k)} \|z\|,$$
$$x^{k+1} = x^k - \alpha z^k,$$

Using generic convergence conditions, based on technique of [2, 3] we study different norm combinations, choice of norms for image exploration problems, as well as constant estimation issues related with norm choice.

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[1]This work was supported by Russian Science Foundation, project 16-11-10015.

An Object Oriented Library for Computing the Range of the Objective Function

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The problem of the efficient computation of the range of the objective function frequently arises in the global deterministic optimization. There are many ways to evaluate such bounds: interval analysis methods, Lipschitz estimators, concave minorants, and many others. These estimates are often implemented by developers manually, which significantly increases the development time and also increases the likelihood of errors in code. In this paper we address this issue by developing the C++ library that automates the computation of function bounds.

The main idea of the proposed approach is to use a single description of a function for a subsequent automatic calculation of the value of the objective function at a given point, derivatives, and interval estimates. The description of a function is constructed with the help of overloaded C++ operators and looks very similar to its algebraic representation. The tool includes the following modules: the mathematical expressions module, the objective function calculation module, the interval analysis module,the gradient calculation module, the interval estimation of gradient module, the Hessian matrix calculation module.

With the help of the developed library the test suite of 150 test optimization problems has been implemented. All functions are implemented by employing the mathematical expressions module. The test suite can be used in testing and comparing global optimization methods.

On the approach to set covering with interval uncertainties in weights of sets

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The set cover problem with interval uncerainties in weights of sets is under consideration. This problem is naturally arises when we deal with measurements errors or variability of parameters in applied problems. The set cover problem with exact values of weights states as follows. Let $U = \{1, \ldots, m\}$, $S = \{S_1, \ldots, S_n\}$, $S_i \subseteq U$, and let $w : S \to \mathbb{R}_+$ be an additive weight function. A cover of U is a such collection $C = \{S_{i_1}, \ldots, S_{i_k}\}$, $S_{i_j} \in S$, that $U = \cup_{j=1}^{k} S_{i_j}$. We need to find an optimal cover, i.e. the cover with minimal weight $w(C) = \sum_{j=1}^{k} w(S_{i_j})$. To find an optimal cover is *NP*-hard problem for exact values of sets' weights [1]. In the sence of guaranteed accuracy, the greedy algorithm [2] is assimptotically the best polynomial algorithm for obtaining of approximate solution of the problem [3].

Using the greedy algorithm, we build an approach to deal with the set cover problem with interval uncertainties in weights of sets. As a result of implementation of the approach, we have the set of approximate solutions of the problem for all combinations of possible weights. We estimate weights of these solutions. If there is some probability distribution that specified on weights' intervals, we compute probabilities of the solutions. The probability of a solution is a probability that we shall have such a combination of possible exact values of sets' weights that the greedy algorithm will give this solution as an output. As an example of the approach application, we use it to form the set of train's runs and to compute their possible costs.

We show that computational complexity of numerical realization of the approach is non-decreasing step function of intervals' widths. It is shown, that even for small values of m, the interval modification of the set cover problem may be computationally hard to solve. We consider ways of modification of such individual problems that the modified problems are numerically solvable and yet they are the closest problems to the initial ones according to some criteria.

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The Problem of Optimal Location of the Foodservice Point in the University Building

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We study a problem of optimal location of the cafeteria in the university building. The building is described as an undirected graph. Vertices of the graph are corresponded to the classrooms, to existing foodservice points and to potential spots of new cafeteria. Edges of the graph describe the connection of the vertices and are characterized by the time of transition between the vertices. Capacity of the classrooms allows us to assess the demand for food. According to the results of the empirical observation and the polls 1) we constructed the utility function on which buyers choose the foodservice point; 2) we determined the desired composition of the menu. Value of the utility function depends on the remoteness of foodservice point from the classrooms, product price, service queue, range of dishes. The potential spots of new cafeteria are characterized with their own costs on opening and rent. The best spot for cafeteria considering incomes is required to be found. This problem is formalized as an optimization problem with equilibrium constraints. As subproblem, we study the problem of complete ration selection for the students in the case of the limited budget and minimal eating time. In the report we study an experience of modeling the problem on the example of the School of Natural Sciences in Far Eastern Federal University.

Optimization of Ambulance Routes

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We study a problem of optimal assignment of ambulance teams on calls maximizing the number of served calls on a given time horizon. At the beginning of period are known: 1) the number of working ambulance teams, their location in the nodes of transportation network and the time until the end of service; 2) the number of calls for the further service, their location in the nodes of network and duration of service. For the planning period are known: 1) loading of the network with traffic flows; 2) the intensity of new calls, their distribution among the different zones of network and estimation of service duration. The calls are classified by the nosological forms and by the priority of service. Each ambulance team is specialized on the certain amount of nosological forms. It is required to assign the teams on calls waiting for service and to identify the route of their movement to maximize the amount of served calls on a given time horizon. This problem is formalized as the integer programming model. In the report we will show an experience of modeling the problem on working ambulance stations of Vladivostok.

Interval regularization for systems of linear algebraic equations

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The present work is devoted to the solution of ill-conditioned systems of linear algebraic equations. We propose a new regularization method based on ideas and methods of interval analysis.

Solving systems of linear algebraic equations of the form

$$Ax = b,$$

with a matrix A and right-hand side vector b, is one of the most important components of many modern computational technologies. However, if the matrix A is ill-conditioned (almost singular), then finding a reliable solution to such a system presents considerable difficulties.

To improve the stability of the solution, we "intervalize" the system $Ax = b$, i.e., we replace it by an interval system $\boldsymbol{A}x = \boldsymbol{b}$ of linear algebraic equations with a matrix $\boldsymbol{A}$ whose elements are obtained by blowing up the elements from A, and the right-hand side $\boldsymbol{b}$ is obtained from the vector b by the similar procedure. As a result, the "inflated" matrix of the system acquires close well-conditioned matrices, for which the solution of the corresponding systems is more stable.

As a pseudo-solution of the original system of linear algebraic equations, a point is taken from the *tolerable solution set* to the intervalized linear system $\boldsymbol{A}x = \boldsymbol{b}$. The rationale is that the tolerable solution set (see [1, 2]), defined as

$$\varXi_{tol} = \big\{ \, x \mid (\forall A \in \boldsymbol{A})(\exists b \in \boldsymbol{b})(Ax = b) \, \big\},$$

is the intersection of partial solution sets corresponding to separate point matrices A from $\boldsymbol{A}$, thus being the most stable among the solution sets to the interval linear system $\boldsymbol{A}x = \boldsymbol{b}$.

To find a point from $\varXi_{tol}$, one can apply either the subdifferential Newton method [2] or the technique based on the use of the so-called recognizing functional [1, 2]. The latter option leads to a non-smooth convex optimization problem, for which efficient numerical methods have been elaborated recently.

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On the regularization for improper problems of convex programming

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Consider the problem of convex programming (CP)

$$\min\{f_0(x) \mid x \in X\}, \tag{1}$$

where $X = \{x \mid f(x) \leq 0\}$, $f(x) = [f_1(x), \ldots, f_m(x)]$, the functions $f_i(x)$ are convex in $\mathbb{R}^n$ for $i = 0, 1, \ldots, m$. We consider problem (1) with inconsistent constraints: $X = \varnothing$. Such models form [1] a very important class of improper problems (IP) of mathematical programming. Let $X_\xi = \{x \mid f(x) \leq \xi\}$, $E = \{\xi \in \mathbb{R}^m_+ \mid X_\xi \neq \varnothing\}$ and $\bar{\xi}_p = \arg\min\{\|\xi\|_p \mid \xi \in E\}$, $\|\cdot\|_p$ denotes p-norm in $\mathbb{R}^m$.

Along with (1), we consider the problem

$$\min\{f_0(x) \mid x \in X_{\bar{\xi}_p}\}. \tag{2}$$

If $X \neq \varnothing$ in problem (1) then we have $\bar{\xi}_p = 0$ and problems (1) and (2) coincide. Otherwise, (2) is an example of possible correction for IP (1), and we may accept the solution of (2) as generalized (approximative) solution of IP (1).

To problem (2) we assign the auxiliary problem

$$\min_x\{F_\alpha(x, r) = f_0(x) + \alpha\|x\|_2^2 + r\|f^+(x)\|_p,\ r > 0\}. \tag{3}$$

The function $F_\alpha(x, r)$ is strongly convex with respect to $x \in \mathbb{R}^n$. Hence, problem (3) has a unique solution for every $r > 0$ and $\alpha > 0$, including the case $X = \varnothing$, in contrast with problem (2). Therefore, the function $F_\alpha(x, r)$ can be used for the analysis of IPCP.

In this report we obtain the estimates characterising the convergence of $F_\alpha(x, r)$ minimizer to approximate solution of the IP. Here, we consider special the situations as follows

a) in problems (1) and (2) instead of the functions $f_i(x)$ some approximations $f_i^\varepsilon(x)$ are known;

b) in problems (2) the different norms for $p = 1, 2, \infty$ are chosen;

c) in problems (2) the vector $\bar{\xi}_p$ is undefinable;

d) the problems (2) is unsolvable.

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Optimality Conditions for General D.C. Constrained Problem

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We consider the general nonconvex problem with the goal function and equality and inequality constraints given by d.c. functions.

We reduce this problem to a problem without constraints by the exact penalty approach and present the penalized problem as a d.c. minimization one, i.e. with d.c. function which is nonsmooth. Furthermore, relations between the original problem and the penalized problem are investigated.

In addition, employing the d.c. structure of penalized problem the Global Optimality Conditions (GOCs) [1, 2] are developed and analysed. In particular, we prove that the GOCs possess the constructive property, i.e. when the GOCs are violated, it is possible to find a feasible (in original problem) vector which is better than the point under investigation.

Moreover, it is shown that the point satisfying the GOCs turns out to be a KKT vector in the original problem. It means, that the new GOCs are related to the Classical Optimization Theory.

Besides we establish that the verification of the GOCs consists in a solution of a family of the partially linearized (w.r.t. the basic nonconvexities of the original problem unified by the exact penalty in one function) problem, and consecutive verification of the principal inequality of the GOCs.

The effectiveness of the GOCs is verified by a number of examples in which the GOCs confirm its ability to escape stationary points and local minima with improving the goal function.

Acknowledgments. This work has been supported by the Russian Science Foundation, Project No. 15-11-20015.

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On Local Search for General Optimization Problem with D.C. Constraints

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The paper addresses a general nonconvex problem with the goal function and constraints given by d.c. functions [1].

This problem is reduced to a problem without constraints by the exact penalization approach [2]. Besides, the goal function of the penalized problem can be represented as a d.c. function.

Furthermore, the relations between the original problem and the penalized one are investigated.

In addition, using the linearization with respect to the basic nonconvexity of penalized problem, we develop a new local search method (LSM) [3] consisting in a consecutive solution of a sequence of linearized problems. Besides, some convergence properties of the method are established. In particular, it is shown that a limit point of the sequence produced by the method is a KKT point [4], but, in addition, it possesses some supplementary properties. Thus, such a limit point turns out to be rather stronger than the usual KKT vector.

Furthermore, the relations between an approximate solution to the linearized convex problem and the KKT vector of the original problem are investigated, and several stopping criteria are substantiated.

Besides, the relations among the Lagrange multipliers of the original problem, those ones of the linearized problem, and the value of the penalty parameter are established. Finally, a preliminary computational testing of the LSM developed has been carried out on several test problems taken from literature.

Acknowledgments. This work has been supported by the Russian Science Foundation, Project No. 15-11-20015.

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On optimization approach to solving nonlinear equation systems[1]

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Consider the following nonlinear equation system [1]:

$$f_i(x) = 0, \quad i = 1, 2, \ldots, m, \tag{1}$$

where $f_i \colon \mathbb{R}^n \to \mathbb{R}$, $i = \overline{1,m}$, are d.c. functions [2], which can be represented as a difference of two convex functions. Further, we reduce system (1) to nonsmooth optimization problem (see [1]), where objective function $F(\cdot)$ is also the d.c. function [2]. Furthermore, we consider the following d.c. representation: $F(x) = G(x) - H(x)$, where the function $G(\cdot)$ are nonsmooth and function $H(\cdot)$ are differentiable.

For solving the optimization problem we apply the Global Search Theory [3] based on necessary and sufficient global optimality conditions. Note that global search method includes two principal parts: local search and procedures of improving a critical point $z \in I\!R^n$ (i.e procedures for funding a point $u \in I\!R^n$ such that $F(u) < \zeta$, where $\zeta := F(z)$) provided by a local search method.

To this end for a fixed vector $y \in I\!R^n$ it is necessary to solve the following nonsmooth convex auxiliary (partially linearized) problem (both on every step of the special local search method and on the stage of improving a critical point). In order to perform it, we solve the nonsmooth auxiliary problems via the smooth convex problems, increasing the dimension from n up to $(m + n)$.

The computational experiments were carried out on test problems with dimension up to 100. For solving auxiliary problems we apply existing methods and software (for instance, IBM ILOG CPLEX). In addition, we compare the effectiveness of developed algorithms with rather popular solvers.

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[1]This research is supported by Russian Science Foundation, project No. 15-11-20015.

Nonlinear Covering for Global Optimization

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We consider a nonconvex optimization problem:

$$\begin{cases} \text{maximize} & \varphi(x) \\ \text{subject to} & x \in D, \end{cases} \tag{1}$$

where D is a nonempty compact in $\mathcal{R}^n$ and $\varphi : \mathcal{R}^n \to \mathcal{R}$ is a continuous convex function. Problem (1) is called convex maximization (CM) when $\varphi(x)$ is convex and we call it a piecewise convex maximization problem (PCMP) if $\varphi(x)$ is the pointwise minimum of convex functions $\varphi(x) = \min\{f_1(x), \ldots, f_m(x)\}$.

The Lebesgue set of a function f for $\alpha \in R$ is defined like $\mathcal{L}_f(\alpha) = \{x \mid f(x) \leq \alpha\}$.

A feasible point z is the global maximum for (1) if and only if all points of the domain are no better than z in sense of maximization, in other words:

$$D \subset \mathcal{L}_\varphi(\varphi(z)).$$

In order to present the main idea for solving (1) we give a definition along with an abstract result on an equivalence of problems.

Definition 1. *An open subset C satisfying conditions*

$$C \subset \mathcal{L}_\varphi(\varphi(y)) \text{ and } C \neq int(\mathcal{L}_\varphi(\varphi(y)))$$

is called **a covering set at level** $\varphi(y)$.

Proposition 1. *Let y be a feasible point for* (1) *such that $\varphi(y) = max\{\varphi(x) \mid x \in D\} - \delta$ for some $\delta > 0$. Let also C be a covering set at level $\varphi(y)$. Then the following problem is equivalent to* (1)*:*

$$\begin{cases} \textit{maximize} & \varphi(x) \\ \textit{subject to} & x \in D \setminus C. \end{cases} \tag{CC}$$

The main algorithmic feature now looks like

- to cover the feasible set (the domain) by a union of covering sets.
- if the domain is covered by C totally, then stop and the global optimum is found.
- otherwise, solve problem (CC) for an improvement.

Our objective is to construct an "(union of covering sets)" such that

$$D \subset \text{ (union of covering sets)} \subset \mathcal{L}_\varphi(\varphi(z)).$$

Starting with an initial guess of covering sets, a method bootstraps its way up to ever more accurate "sandwich" approximations to answer "the global optimum" or "improvement". What concerns the covering set, the first that comes to mind, is use balls (spherical set) as a simpler nonlinear shape.

Acknowledgements: This research benefited from the support of the FMJH “Program Gaspard Monge for optimization and operations research”, and from the support from EDF.

Application of the linearization method for solving quantile optimization problems

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Abstract. The quantile optimization problem is studied in the case where random parameters are small. The linearization method is presented which allows us to replace the original nonlinear loss function in the quantile statement by its model linear in random parameters.
Keywords: quantile criterion, kernel of probability measure, linearization method, Portfolio Selection.

The problem under consideration is minimization of the quantile criterion in the general nonlinear formulation. The quantile criterion is defined as a quantile of the given confidence level for the distribution of a nonlinear loss function which depends nonlinearly on the deterministic decision vector and the vector of random parameters.

The vector of random parameters is assumed to be small. This fact is modeled as an element-by-element product of the vector of small deterministic parameters by a vector of random parameters with a given joint distribution.

The following linearization method is discussed in the report. The original loss function is replaced by its linearized model. The linearization is performed by expanding the original function in a Taylor series w.r.t. the random parameter vector in a zero neighborhood. Thus, the original problem reduces to the problem of quantile optimization of a loss function linear in random parameters. The error that arises in such a replacement has the order of the square of the vector norm of small deterministic parameters.

The linearized quantile optimization problem can be reduced under some regularity conditions to an equivalent minimax problem where the maximum is taken w.r.t. realization of random vector over the kernel of the probability measure. The kernel is intersection of all the closed α-confidence half-spaces. For the Cauchy distribution, normal and uniform distributions, the kernel can be constructed analytically. In cases where such a construction can be hardly found we use an external approximation of the kernel by a polyhedron with a given number of vertices. Two algorithms are presented for this aim.

Let us consider the practically important case where the original loss function depends linearly on the decision vector and the feasible decision set is a polyhedron. Such a model is typical for Portfolio Selection. In this case the above-mentioned minimax problem where the kernel is replaced by its approximating polyhedron can be reduced to the equivalent LP problem of large dimension. The corresponding example for selection of Russian bonds portfolio is discussed.

Acknowledgements. The research was supported by the Russian Science Foundation grant 15-11-10009.

A parallel p-median clustering algorithm

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In this report we address a cluster analysis problem in a sense of a famous discrete facility location problem has been in focus of many researchers for more than 50 years. Given a set $I = \{1, \ldots, m\}$ of potential sites for locating $p \leq m$ facilities, a set $J = \{1, \ldots, n\}$ of customers to be served from open facilities, and d_{ij} defining the distances (transportation costs) of serving customer $j \in J$ from the facility $i \in I$. The p-median problem consists in locating p facilities such that the overall sum of distances from each customer to its closest facility is minimized.

The problem has also become popular due to its broad applicability. Maybe one of the most important application of the p-median problem is clustering [1]. Though the p-median problem is often able to provide competitive and high quality solutions to the cluster analysis problem [2], they do not often apply it to large scale datasets due to the absence of effective methods of solving such large p-median problem instances. The most advanced state-of-the-art approaches are able to find exact or good suboptimal solutions to problems on graph with several tens of thousands nodes.

In this paper we develop an improved modification of the sequential approach proposed in [3] for the p-median problem and its parallel implementation. The algorithm is based on finding the sequences of lower and upper bounds for the optimal value by use of a Lagrangean relaxation method with a subgradient column generation and a core selection approach in combination with a simulated annealing. The parallel algorithm is implemented coupling the shared memory (OpenMP) with the message passing (MPI) technology. It allows us to deal with large instances on modern high performance computing clusters. The effectiveness and efficiency of parallel algorithm is tested and compared with the most effective modern methods on a set of test instances taken from the literature using the HPC-cluster "Academician V. M. Matrosov" [4].

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Multicommodity flows model for the Pacific Russia interregional trade

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The paper describes a model of trade flows among the territories of Pacific Russia based on multi-commodity network flow equilibrium approach based on Boyce and others [1] equilibrium modeling approach to simulate interregional multi-product flows within the transport system.

Let z_{ij}^{rm} is an unknown volume of trade of the product of r-th type delivered from the i-th region to the j-th by m-th type of transport called "mode". The transportation network given among regions which is desribed by nodes and arcs of a network. Let $c_l^{(m)}(y)$ is the cost of flow y moving on arc l by m-th mode, and $g(z,t)$ is transportation costs of moving volume z of a product between regions i and j which also depends of the distance t between them.

Then we introduce h_p^m as an unknown total flow along the path p for mode m, L_m is a number of network arcs for mode m, $f_l^{rm} = \sum_{p=1}^{P_m} d_{lp}^m h_p^{rm}$ is a flow along the arc l for mode m where P_m is a number of all possible paths between any pair of regions for mode m. Numbers d_{lp}^m equals to one if for fixed m the arc l is a part of path p otherwise they equal zero.

Equilibrium distribution of flows over the network can be expressed as complementary slackness conditions $h_p^m(\sum_l^{L_m} c_l^m(f_l^m)d_{lp}^m - u_{ij}^m) = 0$, and $z_{ij}^{rm}(g(z_{ij}^{rm}) + u_{ij}^{rm}) = 0$ which reflects the principle of Wardrop for network equilibrium that, firstly, if the flow h_p^m along the path p is not equal to zero i.e. $h_p^m > 0$, then the total cost of flow moving $c_p^m = \sum_{l=1}^{L_m} c_l^m(f_l^m)d_{lp}^m$ on all the paths p are equal to the equilibrium value costs u_{ij}^{rm} which are independent of the path. Secondly, if for some way between the regions i and j total expenses is strictly greater than equilibrium value costs, i.e. $c_p^m > u_{ij}^m$, then all $h_p^m = 0$. All these mean that none of the unloaded paths do not have a lower cost for transportation than c_p^m.

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Constrained Separating Plane Algorithm with Additional Clipping

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Abstract. The constrained separating plane method with additional clippings (SPACLIP-CON) for nonsmooth optimization is proposed in this paper. The method is efficient and widely applicable to nonsmooth constrained optimization problems with convex objective functions. Experimental results for solving large-scale non-smooth problems are provided.
Keywords: Nonsmooth convex optimization, Subgradient methods, Black-box minimization, Separating plane method, SPACLIP, SPACLIP-CON, Large-scale optimization

We consider the following problem of constrained convex nondifferentiable optimization: $\min_{x \in Q} f(x)$, where $f(x)$ is a convex nonsmooth objective function, $x \in \mathbb{R}^n$, $Q = \{x \in \mathbb{R}^n \mid h(x) \leq 0\}$, $h(x)$ is a constraint nonsmooth scalar function.

This article is devoted to the further investigation of separating plane methods [1, 2, 3, 4]. Methods work in the extended conjugate space of subgradients and the Legendre-Fenchel conjugate of $f(x)$ $f^*(g) = \sup_x \{gx - f(x)\}$. The development of SPACLIP-CON method for the constrained problems is based on the idea of introduction of the conical approximation of a non-trivial recession cone that belongs to the epigraph of the $f(x)$ into the inner approximation of the epi f^*.

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Proximal Bundle Method[1] with Discarding Cutting Planes

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Abstract. A minimization method from a class of bundle methods [1] is proposed for solving nonlinear programming problem. This method is characterized by opportunity of discarding cutting planes. Cutting planes are dropped at the moment, when the model of the convex function have a good approximation quality of the epigraph of the objective function. Convergence of the proposed method is proved.

Let $f(x)$ be a convex function defined in an n-dimensional Euclidian space $\mathrm{R_n}$, $\partial f(x)$ be a subdifferential of the function $f(x)$ at the point $x \in \mathrm{R_n}$. Assume that for any $s(x) \in \partial f(x)$, where $x \in \mathrm{R_n}$, the equality $\|s(x)\| \leq L$ is defined.

Suppose $f^* = \min\{f(x) : x \in \mathrm{R_n}\}$, $X^* = \{x \in \mathrm{R_n} : f(x) = f^*\}$, $X^*(\varepsilon) = \{x \in \mathrm{R_n} : f(x) \leq f^* + \varepsilon\}$, where $\varepsilon > 0$. A method is proposed for finding some point from the set $X^*(\varepsilon)$ with the following input parameters: $\hat{x} \in \mathrm{R_n}, \hat{\delta} > 0, \hat{\mu} > 0, \hat{\theta} \in (0, 1)$.

0. Initialize start parameters $k = 1$, $x_k = \hat{x}$.

1. Assign $i = 1$, $x_{k,i} = x_k$, $\hat{f}_{k,i}(y) = f(x_{k,i}) + \langle s_{k,i}, y - x_{k,i}\rangle$, $s_{k,i} \in \partial f(x_{k,i})$.

2. Find a point $x_{k,i+1} = \operatorname{argmin}\{\hat{f}_{k,i}(y) + \frac{\hat{\mu}}{2}\|y - x_k\|^2 : y \in \mathrm{R_n}\}$, and compute a number $\delta_{k,i} = f(x_k) - [\hat{f}_{k,i}(x_{k,i+1} + \frac{\hat{\mu}}{2}\|x_{k,i+1} - x_k\|^2)]$.

3. If $\delta_{k,i} \leq \hat{\delta}$, then iteration process is stopped, and return the point x_k.

4. If $f(x_{k,i+1}) \leq f(x_k) - \hat{\theta}\delta_{k,i}$, then $x_{k+1} = x_{k,i+1}$, $k := k + 1$, and go to Step 1.

5. Choose $s_{k,i+1} \in \partial f(x_{k,i+1})$, assign $\hat{f}_{k,i+1}(y) = \max\{\hat{f}_{k,i}(y), f(x_{k,i+1}) + \langle s_{k,i+1}, y - x_{k,i+1}\rangle\}$, $i := i + 1$, and go to Step 2.

It is obtained that the proposed method is stopped after finite steps. According to this result the following assertion is proved.

Theorem 1. *The proposed method constructs some point $x' \in X^*(\varepsilon)$.*

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Cutting plane method using penalty functions

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We propose a method of solving a convex programming problem. It is based on the ideas of cutting methods (e.g., [1]) and penalty methods (e.g., [2]). The method uses the polyhedral approximation of the feasible set and the auxiliary function's epigraphs that are constructed on the basis of external penalties.

We solve the problem $\min\{f(x) : x \in D\}$, where $f(x)$ — a convex function in R_n and $D \subset R_n$ — convex bounded closed set.

Let $f^* = \min\{f(x) : x \in D\}$, $X^* = \{x \in D : f(x) = f^*\}$,
$\operatorname{epi}(g, G) = \{(x, \gamma) \in R_{n+1} : x \in G, \gamma \geqslant g(x)\}$, where $G \subset R_n$, $g(x)$ — the function defined in R_n, $W(z, Q)$ — the bunch of normalized generally support vectors for the set Q at the point z, $\operatorname{int} Q$ — interior of the set Q, $K = \{0, 1, \ldots\}$.

We set $F_i(x) = f(x) + P_i(x)$, $i \in K$, where $P_i(x)$ —- a penalty function that satisfies the conditions:

$$P_i(x) = 0 \,\forall\, x \in D,\ P_{i+1}(x) \geqslant P_i(x),\ \lim_{i \in K} P_i(x) = +\infty \text{ for all } x \notin D. \tag{1}$$

The proposed method generates a sequence of approximations $\{x_k\}$ as follows. Fix a number $\Delta_0 > 0$, a point $v = (v', \gamma')$, where $v' \in \operatorname{int} D$, $\gamma' > f(v')$ and define a convex penalty function $P_0(x)$ with the condition (1). Construct convex closed sets $M_0 \subset R_{n+1}$ and $D_0 \subset R_n$ such that $\operatorname{epi}(F_0, R_n) \subset M_0$, $D \subset D_0$. Set $\bar{\gamma} \leqslant f_0^*$, where $f_0^* = \min\{f(x) : x \in D_0\}$. Fix $i = 0$, $k = 0$.

1. Find $u_i = (y_i, \gamma_i)$, where $y_i \in R_n$, $\gamma_i \in R_l$, as a solution of the problem

   $$\min\{\gamma : x \in D_i,\ (x, \gamma) \in M_i,\ \gamma \geqslant \bar{\gamma}\}.$$

   If $u_i \in \operatorname{epi}(f, D)$, then $y_i \in X^*$.

2. In some way choose a point $v_i \notin \operatorname{int}\operatorname{epi}(F_i, R_n)$ in the interval (v, u_i). Let $M_{i+1} = M_i \cap \{u \in R_{n+1} : \langle a_i, u - v_i \rangle \leqslant 0\}$, where $a_i \in W(v_i,\ \operatorname{epi}(F_i, R_n))$.

3. Let $D_{i+1} = D_i \cap \{x \in R_n : \langle b_i,\ x - v_i' \rangle \leqslant 0\}$, where $b_i \in W(v_i', D)$,
   $v_i' = (v', y_i) \setminus \operatorname{int} D$, Or $D_{i+1} = D_i$.

4. If $F_i(y_i) - \gamma_i > \Delta_k$, than set $P_{i+1}(x) = P_i(x)$ and go step 5. Otherwise choose convex penalty function $P_{i+1}(x)$ satisfying the given conditions (1) and set $x_k = y_i$, $\sigma_k = \gamma_i$. Choose $\Delta_{k+1} > 0$ and go to step 5 with the value of k increased by one.

5. Increase the value of i by one and go to step 1.

Let's note that it is possible to set $\Delta_k = \Delta > 0$, for all $k \in K$, and, in particular, suppose that Δ is arbitrarily large or set $\Delta_k \to 0$, $k \to \infty$, $k \in K$.

It is proved that for every limit point $(\bar{x}, \bar{\sigma})$ of the sequence $\{(x_k, \sigma_k)\}$ the following equalities hold:

$$\bar{x} \in X^*, \quad \bar{\sigma} \in f^*.$$

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A Variant of the Cutting-Plane Method without Nested Embedding Sets

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Abstract. We propose a variant of the cutting-plane method [1] with approximating an epigraph of the objective function which is characterized by possibility of updating embedding sets due to dropping cutting planes. Updates occur on the basis of the constructed criterion of approximation quality for the epigraph by polyhedron sets. Convergence of the method is proved and discuss its implementations.

A problem is solved for minimizing convex function $f(x)$ on the convex closed set $D \subset \mathrm{R_n}$. Suppose that x^* is a solution, $f^* = f(x^*)$, $\mathrm{epi}\, f(x) = \{(x,\gamma) \in \mathrm{R_{n+1}} : x \in \mathrm{R_n}, f(x) \le \gamma\}$, $K = \{0, 1, \dots\}$.

The method is proposed as follows. Choose a point $v \in \mathrm{int}\,\mathrm{epi} f(x)$, convex closed sets $M_0 \subseteq \mathrm{R_{n+1}}$, $G_0 \subset \mathrm{R_n}$ such that $\mathrm{epi} f(x) \subset M_0$, $x^* \in G_0$. Select numbers $\bar{\gamma}$, λ_k, τ_k, $k \in K$ according to conditions $\bar{\gamma} \le f^*$, $\lambda_k \in (0,1)$, $\tau_k \ge 0$, $\lambda_k \to 0$, $k \to \infty$, $\tau_k \to 0$, $k \to \infty$. Assign $D_0 = D \bigcap G_0$, $i = 0$, $k = 0$.

1. Find a solution $u_i = (y_i, \gamma_i)$, where $y_i \in \mathrm{R_n}$, $\gamma_i \in \mathrm{R_1}$, of the problem $\min\{\gamma : x \in D_i, (x,\gamma) \in M_i, \gamma \ge \bar{\gamma}\}$. A point $\bar{u}_i \notin \mathrm{int}\,\mathrm{epi} f(x)$ is selected in the interval $(v, u_i]$ such that $u_i + q_i(\bar{u}_i - u_i) \in \mathrm{epi} f(x)$ for some $1 \le q_i \le q < +\infty$. If $\bar{u}_i = u_i$, then y_i is a solution of the initial problem. Otherwise, choose a bounded set A_i of generally supported vectors for the set $\mathrm{epi} f(x)$ at the point $\bar{u}_i$.

2. If $\|\bar{u}_i - u_i\| > \lambda_k \|v - u_i\|$, then assign $M_{i+1} = M_i \bigcap \{u \in \mathrm{R_{n+1}} : \langle a, u - \bar{u}_i \rangle \le 0 \ \forall a \in A_i\}$ and go to Step 4. Otherwise, go to Step 3.

3. Choose a point $x_k \in D_i$ such that $f(x_k) \le f(y_i) + \tau_k$, assign $\delta_k = \gamma_i$, $M_{i+1} = M_{r_i} \bigcap \{u \in \mathrm{R_{n+1}} : \langle a, u - \bar{u}_i \rangle \le 0 \ \forall a \in A_i\}$, where $0 \le r_i \le i$, $k := k + 1$.

4. Choose a convex closed set $G_{i+1} \subset G_0$ according to condition $x^* \in G_{i+1}$, assign $D_{i+1} = D \bigcap G_{i+1}$, increment i by one, and go to Step 1.

Optimal criterion from Step 1 is proved. It is obtained that sequences $\{x_k\}$, $\{\delta_k\}$ will be constructed together with the sequence $\{u_i\}$.

Theorem 1. *For constructed sequences $\{x_k\}$, $\{\delta_k\}$ it is obtained that* $\lim\limits_{k\to\infty} f(x_k) = \lim\limits_{k\to\infty} \delta_k = f^*$.

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Dual Algorithms for Linear Semidefinite Optimization

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In this paper we consider dual affine-scaling and simplex-like algorithms for linear semidefinite optimization. The main attention will be given to the combined algorithms with both properties of affine-scaling and simplex-like methods.

Let $\mathbb{S}^n$ denote the space of real symmetric matrices of order n, and let $\mathbb{S}^n_+$ denote the cone of positively semidefinite matrices from $\mathbb{S}^n$. The scalar (inner) product $M \bullet M_2$ of two matrices M_1 and M_2 from $\mathbb{S}^n$ is defined as the trace of the matrix $M_1 M_2$.

The linear semi-definite programming problem is to find

$$\min_{X \in \mathcal{F}_P} C \bullet X, \qquad \mathcal{F}_P = \left\{ X \in \mathbb{S}^n_+ : \; A_i \bullet X = b^i, \quad i = 1, \dots, m \right\}, \tag{1}$$

where $C \in \mathbb{S}^n$ and $A_i \in \mathbb{S}^n$, $1 \leq i \leq m$, are given. The dual problem to (1) has the form

$$\max_{u \in \mathcal{F}_D} b^T u, \qquad \mathcal{F}_D = \left\{ u \in \mathbb{R}[m] : \; V(u) = C - \sum_{i=1}^{m} u^i A_i \in \mathbb{S}^n_+ \right\}, \tag{2}$$

where $b = [b^1, \dots, b^m]$. We assume that the matrices A_i, $1 \leq i \leq m$, are linear independent. We assume also that problems (2) is nondegenerate, i.e. all feasible points from $\mathcal{F}_D$ are nondegenerate.

Let X_* and $[u_*, V_*]$ with $V_* = V(u_*)$ be the solutions of problems (1) and (2) respectively. In what follows we assume that the strict complementary condition holds in $[X_*, V_*]$.

At first consider the simplex-like algorithm. Denote by $\mathcal{E}(\mathcal{F}_D)$ the subset of extreme points of the feasible set $\mathcal{F}_D$. Starting from the extreme point u_0 the algorithm generates the sequence of extreme points $\{u_k\} \subset \mathcal{E}(\mathcal{F}_D)$ which converges to u_*. The optimality conditions for both problems (1) and (2) are used essentially to make pivoting at each step of the algorithm.

The second algorithm is the combination of the affine-scaling and simplex-like algorithms. At each iteration the search direction Δu in this algorithm is the linear combination of two directions, namely, $\Delta u = \Delta u^1 + \tau \Delta u^2$, where τ is a positive coefficient. The direction Δu^1 belongs to the minimal face of the set $\mathcal{F}_D$, containing the current point u. This direction is defined in the usual way by means of approach using in dual affine-scaling algorithms. The second direction Δu^2 gives us possibility to jump from one boundary face to another one. In the case where the minimal face is an extreme point of $\mathcal{F}_D$, the method can behave as the simplex method. Both algorithms converge to the solution u_* of the dual problem (2). Simultaneously we obtain the solution X_* of the problem (1).

DISCRETE OPTIMIZATION

Approximating the 2-Machine Flow Shop Problem with Exact Delays Taking Two Values

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We study a special case of the 2-Machine Flow Shop problem with exact delays. An instance of the problem consists of n triples (a_j, l_j, b_j) of nonnegative integers where j is a job in the set of jobs $\{1, \ldots, n\}$. Each job j must be processed first on machine 1 and then on machine 2, a_j and b_j are the lengths of operations on machines 1 and 2, respectively. The operation of job j on machine 2 must start exactly l_j time units after the operation on machine 2 has been completed. The goal is to minimize makespan.

The approximability of the general case was studied by Ageev and Kononov in [1]. They proved that the existence of $(1.5-\varepsilon)$-approximation algorithm implies P=NP and constructed a 3-approximation algorithm. In this paper we consider the case when $l_j \in \{0, L\}$ for all $j \in \{1, \ldots, n\}$. In the standard three-field notation scheme this case can be written as $F2 \mid \text{exact } l_j \in \{0, L\} \mid C_{\max}$. The problem includes as a special case the classical no-wait 2-Machine Flow Shop problem which is known to be solvable in polynomial time [2]. Our results are the following: we prove that the existence of $(1.25-\varepsilon)$-approximation for $F2 \mid \text{exact } l_j \in \{0, L\} \mid C_{\max}$ implies P=NP and present a 2-approximation algorithm.

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Approximation-Preserving Reduction of k-Means Clustering with a Given Center to k-Means Clustering

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We consider the following clustering problem introduced by A. Kelmanov and studied in a series of papers by him and his coauthors (see [1] for the references).

Problem k-Means+Fixed Center.

Instance: A set X consisting of n points $\mathbb{R}^d$ and a positive integer k.

Goal: Find a family of mutually disjoint subsets $C_1, \dots C_k \subseteq X$ minimizing the function

$$\sum_{i=1}^{k} \sum_{x \in C_i} \|x - z(C_i)\|^2 + \sum_{x \in X \setminus (\bigcup_i C_i)} \|x\|^2$$

where $z(C_i)$ stands for the centroid of C_i.

Problem k-Means+Fixed Center is a natural modification of the classical k-Means problem:

Problem k-Means.

Instance: A set X consisting of n points $\mathbb{R}^d$ and a positive integer k.

Goal: Find a partition $C_1, \dots C_k$ of X minimizing the function

$$\Psi(\mathcal{C}) = \sum_{i=1}^{k} \sum_{x \in C_i} \|x - z(C_i)\|^2$$

Theorem 1. *Let $\mathcal{A}$ be an algorithm solving k-means with approximation ratio $1 + \delta$ and time complexity $T(d, n, k, 1/\delta)$. Then problem k-means+fixed center can be solved with approximation ratio $(1 + \varepsilon)(1 + \delta)$ in time $T(d, n(1 + 1/\varepsilon), k + 1, 1/\delta)$.*

This theorem allows to carry known algorithmic results from k-Means problem to k-Means+Fixed Center problem.

Acknowledgments. This work was supported by the Russian Foundation for Basic Research (project 15-01-00462).

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Complexity of Normalized K-Means Clustering Problems

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We study the computational complexity of the following two clustering problems.

Problem 1 (*Normalized K-Means Clustering*). Given a set $\mathcal{Y}$ of N points in $\mathbb{R}^d$ and a positive ineger $K \geq 2$, find a partition of $\mathcal{Y}$ into clusters $\mathcal{C}_1, \ldots, \mathcal{C}_K$ minimizing

$$\sum_{k=1}^{K} \frac{1}{|\mathcal{C}_k| - 1} \sum_{y \in \mathcal{C}_k} \|y - \overline{y}(\mathcal{C}_k)\|^2$$

where $\overline{y}(\mathcal{C}_k)$ is a centroid of cluster $\mathcal{C}_k$.

Problem 2 (*Normalized K-Means clustering with a given center*). Given a set $\mathcal{Y}$ of N points in $\mathbb{R}^d$ and a positive ineger $K \geq 2$, find a partition of $\mathcal{Y}$ into clusters $\mathcal{C}_1, \ldots, \mathcal{C}_K$ minimizing

$$\sum_{k=1}^{K-1} \frac{1}{|\mathcal{C}_k| - 1} \sum_{y \in \mathcal{C}_k} \|y - \overline{y}(\mathcal{C}_k)\|^2 + \frac{1}{|\mathcal{C}_K| - 1} \sum_{y \in \mathcal{C}_K} \|y\|^2$$

where $\overline{y}(\mathcal{C}_k)$ is a centroid of cluster $\mathcal{C}_k$.

The problems are important, in particular, in applied statistics, data mining and machine learning.

The complexity status of the problems seemed to be unclear up to now. In this paper we prove that Problem 1 is strongly NP-hard for each fixed $K \geq 3$ and Problem 2 is strongly NP-hard for each fixed $K \geq 4$.

Acknowledgments. This work was supported by the Russian Foundation for Basic Research (project 15-01-00462).

A parameterized approximation algorithm for the mixed capacitated arc routing problem: Theory and experiments

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The mixed capacitated arc routing problem (MCARP) models the task of finding minimum-cost tours for servicing links in transportation networks using a fleet of vehicles with equal capacity. Herein, *mixed* means that the transportation network may have undirected edges and directed arcs.

Problem (MCARP).
Instance: A mixed graph $G = (V, E, A)$ with edge set E and arc set A, a depot vertex $v_0 \in V$, traversal costs $c : E \cup A \to N \cup \{0\}$, demands $d : E \cup A \to N \cup \{0\}$, and a vehicle capacity Q.
Goal: Find a set W of closed walks in G, each passing through the depot vertex v_0, and a *serving function* $s : W \to 2^{E \cup A}$ determining for each walk $w \in W$ the subset $s(w)$ of the edges and arcs it *serves*, such that

- $\sum_{w \in W} c(w)$ is minimized, where $c(w) := \sum_{i=1}^{\ell} (e_i)$ for $w = (e_1, e_2, \ldots, e_\ell)$,
- $\sum_{e \in s(w)} d(e) \leq Q$, and
- each edge or arc e with $d(e) > 0$ is served by exactly one walk in W.

If $A = \emptyset$, then MCARP is $(7/2 - 3/W)$-approximable. Otherwise, even if $E = \emptyset$, MCARP is as least as hard to approximate as the n-vertex metric asymmetric Traveling Salesperson problem ($\Delta - ATSP$), for which the best known is a polynomial-time $O(\log n / \log\log n)$-approximation. Conversely, we prove:

Theorem (van Bevern, Komusiewicz, Sorge [1]). Any polynomial-time $\alpha(n)$-approximation algorithm for the n-vertex $\Delta - ATSP$ yields a polynomial-time $O(\alpha(C))$-approximation algorithm for MCARP, where C is the number of weakly connected components in the graph induced by positive-demand arcs and edges.

The number C is small in many applications and benchmark data sets (where, usually, $C = 1$). We present some heuristic enhancements that help our algorithm to outperform many previous polynomial-time heuristics.

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[1]Supported by RFBR grant 16-31-60007 mol_a_dk.

The problem of processing total time minimization for identical workpieces

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Abstract. We consider the problem of processing identical workpieces with complicated technological route. The problem of processing total time minimization for identical workpieces is NP-hard in cases of four or more machines.

Keywords: identical workpieces, complexity, processing total time.

It is required to process a batch of N identical workpieces on a production line [1]. All workpieces are processed by the same technological route that consists of n successively performed operations. There are machine and processing time are specified for every operation. Simultaneous carrying out two or more operations on one machine is nor allowed. As usual, classical conveyer can be used. However, the rapidly changing range of products requires a renovation of the production line, which leads to the introduction of high-priced universal workstations capable of performing many different operations. A workpiece can get to these machines repeatedly in its processing. Similar situation arises in the assembly of aircrafts and rocket launchers, when some team repeatedly comes back to the worksite to develop next operation according to their profile. Asymptotically exact algorithms of minimization of the total processing time of identical workpieces based on cyclic schedules are described in [2, 3].

The problem of processing total time minimization for identical workpieces is NP-hard in cases of four or more machines. This complexity can be proofed by reducing of the NP-hard Job Shop problem with three workpieces and three machines [4] to problem with several identical workpieces.

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On the skeleton of the pyramidal tours polytope[1]

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The skeleton of the polytope P is the graph whose vertex set is the vertex set of P and edge set is the set of 1-faces of P. There are two results on the traveling salesman polytope $\mathrm{TSP}(n)$ of interest to us: the question whether two vertices of the $\mathrm{TSP}(n)$ are nonadjacent is NP-complete [4], and the clique number of the $\mathrm{TSP}(n)$ skeleton is superpolynomial in dimension [1]. It is known that this value characterizes the time complexity in a broad class of algorithms based on linear comparisons [2].

Hamiltonian tour is called a pyramidal if the salesman starts in city 1, then visits some cities in increasing order, reaches city n and returns to city 1 visiting the remaining cities in decreasing order. Pyramidal tours have two nice properties. First, a minimum cost pyramidal tour can be determined in $O(n^2)$ time by dynamic programming. Second, there exist certain combinatorial structures of distance matrices that guarantee the existence of a shortest tour that is pyramidal [3].

We consider the skeleton of the pyramidal tours polytope $\mathrm{PYR}(n)$ that is defined as the convex hull of characteristic vectors of all pyramidal tours in the complete graph K_n. We describe necessary and sufficient condition for the adjacency of the $\mathrm{PYR}(n)$ polytope vertices. Based on that, we establish following properties of the $\mathrm{PYR}(n)$ skeleton.

Theorem 1. *The question whether two vertices of the* $\mathrm{PYR}(n)$ *are adjacent can be verified in linear time* $O(n)$.

Theorem 2. *The clique number of the* $\mathrm{PYR}(n)$ *skeleton is* $\Theta(n^2)$.

Thus, the clique number correlates with the time complexity $O(n^2)$ of dynamic programming for pyramidal traveling salesman problem.

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[1]The research is supported by the initiative R&D VIP-004 YSU

Approximate Solution of Length-Bounded Maximum Multicommodity Flow with Unit Edge-Lengths[1]

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We propose an improved fully polynomial-time approximation scheme (FPTAS) and a greedy heuristic for the fractional length-bounded maximum multicommodity flow problem with unit edge-lengths [1]. The proposed FPTAS has a lower time complexity bound compared to the previously known algorithm [2] designed for a problem with the length functions of more general form.

Computational experiments are carried out on benchmark graphs and on graphs that model software defined satellite networks to compare the proposed algorithms and an exact CPLEX LP solver. The results of experiments demonstrate a trade-off between the computing time and the precision of algorithms under consideration. The FPTAS and greedy heuristic are significantly faster than the CPLEX LP solver, especially on the instances with large networks and great numbers of demands. The FPTAS is more accurate but requires greater CPU time than greedy heuristic, which may be a decisive factor in practical applications.

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[1]This research is supported by the Russian Science Foundation grant 15-11-10009.

On the optima localization for the 2-machine routing open shop with unrelated travel times

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The routing open shop model first introduced in [1, 2] is considered. In this model the sets of jobs and machines are given and each machine has to perform operations of each job (with given processing times) in arbitrary order similar to the classic open shop scheduling problem. Jobs are distributed among the nodes of some transportation network represented by edge-weighted graph. Weight of edge represents travel distance between the correspondent nodes and. All the machines are initially located at a predefined special node referred to as *the depot* and have to travel with unit speed between the nodes of transportation network to process their operations. The goal is to minimize the makespan which is the time moment of returning of the last machine to the depot after processing all the operations. This problem is proved to be NP-hard even in the simplest non-trivial case with two machines on a link [1].

The standard lower bound $\bar{R}$ for the optimum of the routing open shop was introduced in [2]. It was proved that optima for any instance with two machines and two nodes always belongs to the interval $[\bar{R}, \frac{6}{5}\bar{R}]$ and the bounds are tight. This result was generalized on the case of three nodes in [4].

In this paper we consider the generalization of the two machine routing open shop with individual machines' travel times [3]. We prove that for cases of two and three nodes with both proportional and unrelated travel times optima for any problem instance belongs to the interval $[\bar{R}, \frac{5}{4}\bar{R}]$ and the bounds of the interval are tight.

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On the abnormality in open shop scheduling

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We consider the classic open shop scheduling problem to minimize the makespan [1]. An input I of the problem can be described by an matrix of processing time $P = (p_{ji})_{m\times n}$, m and n being the numbers of machines and jobs respectively. The *standard lower bound* for instance I is defined as $\bar{C}(I) \doteq \max\left\{\max\limits_i \sum\limits_j p_{ji}, \max\limits_j \sum\limits_i p_{ji}\right\}$. Let us denote *the total load* of instance I by $\Delta(I) \doteq \sum\limits_{i,j} p_{ji}$. Note that by definition $\Delta(I) \leqslant m\bar{C}(I)$.

A feasible schedule S for instance I is referred to as *normal* if its makespan $C_{\max}(S)$ coincides with $\bar{C}(I)$ [2]. An instance I is *normal* if a normal schedule for I exists. It is well known that any two-machine open shop instance is normal [1] while for $m \geqslant 3$ that is not the case.

For any instance I we define its *abnormality* as $\alpha(I) \doteq C^*_{\max}(I)/\bar{C}(I)$, where $C^*_{\max}(I)$ is the makespan of optimal schedule for I. The natural question is, how large can an abnormality of some instance be.

It was shown in [3] that the maximal abnormality for any three-machine open shop instance if equal to $\frac{4}{3}$. That value is achieved on the instance I' with the following matrix of processing times $\left(\begin{smallmatrix}1&2&0&0\\1&0&2&0\\1&0&0&2\end{smallmatrix}\right)$. Note that the total load of I' reaches an extremal value of $3\bar{C}(I')$.

In this paper we discuss the maximal abnormality for the three-machine open shop as a function of the total load (specifying our previous result from [3]). More precisely, let $\mathcal{I}_m(x) \doteq \{I \text{ is an instance of } m\text{-machine open shop} | \Delta(I) \leqslant x\bar{C}(I)\}$. Then we consider the following *abnormality function* $F_m(x) \doteq \sup\limits_{I\in\mathcal{I}_m(x)} \alpha(I)$. We show that $\forall m \leqslant 2\, \forall x \in [1, 2]\ F_m(x) = 1$ and $\forall x \geqslant 2\ F_m(x) \leqslant x/2$, and describe "almost exact" form of function $F_3(x)$.

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Upper and lower bounds for the leader-follower facility location game under budget constraints

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We study a new facility location game. Two players, a leader and a follower, open facilities and compete to attract clients on a given market. Each player has a budget and try to maximize own market share. They have to pay for opening facilities and its attractiveness for the clients. Utility of each facility for client is directly proportional to the attractiveness and inversely proportional to the distance between client and facility. Each client patronizes exactly one facility with the best utility. In case of ties, the follower's facility is preferred. The goal of the game is to find location of the leader's facilities and their attractiveness to maximize leader's market share. We present this game as a mixed integer bi-level linear program and show its computational complexity. To get lower bound for the global optimum, we design a stochastic local search matheuristic with alternating neighborhoods. Optimal solution for one player is calculated for the fixed solution for another player by CPLEX software. Upper bound is obtained by the following framework. We rewrite the bi-level problem as a single level problem with exponential set of constraints and variables. Exact solution to this problem with a subset of constraints and variables produces an upper bound. For improving the bound, we iteratively enlarge this subset. Computational results for 100 potential facilities are discussed.

This research was supported by RSF grant 15-11-10009.

A matheuristic for the $(r|p)$-centriod problem under L1 metric

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We consider the $(r|p)$-centroid problem under L1 metric. This Stackelberg game was first studied by Hakimi in 1981 [1] for location on a network. It's a well-known bi-level facility location problem, in which, two players, called the leader and the follower, open facilities to service clients. We assume that clients are identified with their location on the 2-dimensional plane, and facilities can be opened anywhere in the plane. The leader opens p facilities. Later on, the follower opens r facilities. Each client patronizes the closest facility. The distance between the clients and facilities is determined according to the $L1$ metric. In case of ties, the leader's facility is preferred. The goal is to find p facilities for the leader to maximize his market share.

While this problem is well studied in case of Euclidean metric, e.g. [2] and more , it's not the case for the Manhattan version. In this work we provide the complexity results concerning both the Follower's and the Leader's problem. In order to tackle the problem we propose an effective heuristic method combined with the mathematical programming techniques (a matheuristic). We discuss the results of the numerical experiments which was carried out on instances form the benchmark library "Discrete Location Problems" [3]

Acknowledgements. This research is supported by the Russian Science Foundation grant 15-11-10009.

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Genetic Algorithm with Optimal Recombination for Makespan Minimization on Single Machine

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The strongly NP-hard Makespan Minimization Problem on a Single Machine ($1|l_{ij}|C_{\max}$) [2] is considered. We propose a new genetic algorithm (GA) with steady state scheme for $1|l_{ij}|C_{\max}$, which solves an *Optimal Recombination Problem* (ORP) within crossover operator. Given two parent solutions, the ORP consists in finding the best possible offspring as a result of a crossover operator, obeying the conditions of gene transmitting recombination [3].

The computational complexity and solving method of the ORP for $1|l_{ij}|C_{\max}$ has been analyzed in [1]. Moreover, we have investigated in [1] two simple crossover-based GAs using ORPs but no local search procedures or fine-tuning of parameters were employed. In comparison to the GAs from [1], the GA proposed in this paper uses problem-specific local search heuristics and greedy constructive heuristics to generate the initial population. In addition, this GA applies mutation operators and a restart rule to avoid localization of the search and restore the population diversity.

The experimental evaluation shows that the proposed GA with ORP yields competitive results. In particular, the GA demonstrates at least 98% frequency of finding an optimum in the same time as in [1] on all instances generated from ATSP instances of TSPLIB library by assigning setup times for $1|l_{ij}|C_{\max}$ equal to weights of arcs in ATSP. We estimate that the average frequency of obtaining an optimum by the best GA from [1] is by factor 2.6 smaller than such frequency for the GA presented here. The experimental comparison indicates an advantage of the optimized crossover based on solving the ORP over its randomized prototype. We also analyze effectiveness to use local search not only at the initialization stage, but on iterations of our GA with ORP.

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[1]This research is supported by the Russian Science Foundation grant 15-11-10009.

Convergecast Scheduling Problem on a Square Grid with Obstacles

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In the wireless networks the data collected by the devices should be aggregated in the base station (BS). Aggregation time, i.e. a period during which the data from all network elements fall in the BS, is the most important criterion in the quick response networks. In the formulations of the aggregation problem the volume of the transmitted data, as usually, does not taken into account. Each packet is transmitted along any edge of communication graph (CG) during one time round (slot).

In the majority of the wireless networks, an element (vertex or node) cannot transmit and receive packets at the same time (half duplex mode), and a vertex cannot receive more than one packet simultaneously. Moreover, due to the need of energy saving, each sensor sends the packet once during the aggregation period. This means that the packets are transmitted along the edges of a desired *aggregation tree* (AT) rooted in the BS, and an arbitrary vertex in the AT must first receive the packets from all its children, and only then send the aggregated packet to its parent node.

In the most wireless networks, the transmitters use common radio frequency. So, if in the receiver's reception area working more than one transmitter, then (due to the radio wave interference phenomenon), the receiver cannot get any correct data packet. Such a situation is called a *conflict*.

In the conflict-free data aggregation problem it is necessary to find the AT and a conflict-free schedule of minimal length [1]. This problem is known as a Convergecast Scheduling Problem (CSP), and it is NP-hard even in the case when AT is given. In [2] a special case of CG in the form of a unit square grid, in which in each node a transmitter is located, and the transmission range of each element is 1, is considered. A simple polynomial algorithm for constructing an *optimal* solution to this problem was proposed.

In this paper, we consider the similar CG in the form of square grid but with rectangular obstacles and propose a polynomial algorithm for conflict-free data aggregation scheduling.

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Large-scale problems of discrete optimizaton: an asymptotically optimal approach vs a curse of dimensionality

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Keywords: capacitated FLP, exact algorithm, tree network, line graph.

For discrete optimization problems, the main factor determining the feasibility of algorithms for their solution is the dimension (the length of the entry record) of the problem, which in the years 50-70 at the last century was associated with "a curse of dimensionality".

In contrast, within the framework of an asymptotically optimal approach to (approximate) solution of difficult problems of discrete optimization, the dimension of the problem is our friend and confederate.

We are talking about problems such as routing, multi-index assignments, clustering, allocation, extreme problems on graphs and networks, and so on.

Usually, these problems are difficult to solve (NP-hard), which causes the urgency of developing effective approximation algorithms with guaranteed estimates of the quality of their work — time complexity, performance ratio (or relative error), reliability of operation.

The curse of dimensionality is a problem associated with the exponential increase in the time of the solution of the problem due to the increase in the dimensionality of space (Richard Bellman, 1961).

Estimation of the relative error of the algorithm A of the solution on deterministic inputs of size n are called $\varepsilon_A(n)$ such that $|W_A(I) - OPT(I)|/OPT(I) \leq \varepsilon_A(n)$ is true on any input of I, where $OPT(I)$ and $W_A(I)$ are values of the objective functions, optimal and found by the algorithm A at the input I.

For problems on random inputs, the quality of the algorithm is also characterized by the probability of failure $\delta_A(n)$ which equal to the fraction of cases when the algorithm A does not guarantee the solution with the declared error. The better the algorithm, the smaller $\varepsilon_A(n)$ and $\delta_A(n)$.

The first example of an algorithm with an almost always guaranteed relative error is more than half a century ago! (Borovkov A. A. To the probabilistic formulation of two economic problems // DAN SSSR. 1962. 146 (5). 983–986): TSP and AP.

The algorithm A with the estimates ε_n and δ_n is asymptotically optimal if $\varepsilon_A(n) \to 0$ and $\delta_A(n) \to 0$ when $n \to \infty$.

The report presents examples of the implementation of an asymptotically optimal approach to the solution of some large-dimensional problems of discrete optimization in the operations research in which the author has been directly involved in the past half century.

[1]The authors are supported by the Russian Foundation for Basic Research grants 16-07-00168 and 15-01-00976.

On Some Realizations of Solving the Resource Constrained Project Scheduling Problems

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We consider the resource constrained project scheduling problem (RCPSP) with precedence and resource constraints. The RCPSP can be defined as a combinatorial optimization problem, i.e. in terms of decision variables, constraints and objective functions, as follows. A set of activities and a set of resources of known characteristics (activity durations, activity resource demands, resource availabilities, precedence restrictions) are given. The decision variables are the activity start times defined on integer time periods. The objective function which has to be minimized is the makespan, i.e. the largest activity completion time, assuming the project starts at time 0. There are two types of constraints. The precedence constraints prevent each activity from starting before the completion of its predecessors. The resource constraints ensure that, at each time period and for each resource, the total activity demand does not exceed the resource availability. Once started, an activity cannot be interrupted.

The RCPSP belongs to the class of NP-hard optimization problems and is actually one of the most intractable classical problems in practice.

We propose two methods for solving the problem. One of them is the exact branch and bound algorithm. For another, we use metaheuristics – genetic algorithm. Both of the proposed algorithms use the solution of the relaxed problem where all constrained resources being accumulative as an auxiliary problem.

We present results of numerical experiments illustrating quality of proposed algorithm. The test instances were used from the library of test problems PSPLIB. Numerical experiments demonstrated algorithm's competitiveness. We have found the best solutions for a few instances from the dataset j120, and the best average deviation from the critical path lower bound for the datasets j60 (50000 and 500000 iterations) and j120 (500000 iterations).

[1]The authors are supported by the Russian Foundation for Basic Research (project no. 16-07-00829, 15-01-00976, 16-07-00168)

Finding Bounded-below-Diameter Minimum-edge disjoint Spanning Trees

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The classic minimum spanning trees problem is to find k edge-disjoint spanning trees in given undirected weighted graph. It can be solved in polynomial time [1]. In the diameter bounded below minimum spanning trees problem there is an additional requirement: a diameter of every spanning tree must be not less than some predefined value d. The diameter of a tree is the number of edges on the longest path between two leaves in the tree. We will abbreviate this problem as k-MSTBB. The k-MSTBB is NP-hard since for $k = 1$ and $d = n - 1$, where n is the number of vertices in the graph, it is equivalent to the Hamiltonian path problem. The k-MSTBB for $k = 1$ was introduced in [2] and the asymptotically optimal algorithm for the case of uniform distribution on a segment was presented. k-MSTBB for arbitrary k and uniform distribution was studied in [3].

In current work we consider the k-MSTBB on compete n-vertices undirected graphs where edges weights are independent identically distributed random variables with discrete distribution on a set $\{1, \ldots, b_n\}$. We propose a polynomial time algorithm to solve the problem.

The algorithm builds k edge-disjoint Hamiltonian chains with d edges each on the same $d+1$ vertices. Then by the algorithm by Roskind and Tarjan [1] algorithm finds k edge-disjoint spanning trees containing these chains.

A probabilistic analysis was performed under conditions that graph edges weights are identically independent distributed random variables with uniform distribution on a set $\{1, b_n\}$, $0 < b_n < \infty$. Also it is supposed that $d = d_n$ goes to infinity as n goes to infinity. It was shown that proposed algorithm finds optimal solution with high probability (whp), i.e. probability tends to 1 as n goes to ∞.

This research was supported by the Russian Foundation for Basic Research (grants 15-01-00976, 16-07-00168 and 16-31-00389).

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On some effective algorithms for solving capacitated clustering and location problems on the tree and the real line

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Keywords: capacitated FLP, exact algorithm, tree network, line graph.

The network $G = (V, E)$ is considered. At the vertices of the set $V = \{1, \dots, n\}$ there are consumers of some product and possible places of its production. For each vertex $i \in V$, there are given demand volume b_i, the cost g_i^0 for the placement of the facility and the restriction a_i on the facility's capacity. For each edge $e \in E$, there are given the cost transportation of the product unit and the maximum quantity c_e of a product that can be transported along this edge. It is required to find open facilities which satisfy all demand with minimal total cost of opening facilities and delivering consumers.

The report proposes an exact algorithm for solving the problem on a tree network with time complexity $O(nb^2)$ (where $b = \sum_{i=1}^{n} b_i$). We note that in [1] an algorithm is given with the same time complexity, but in the case of unlimited production volumes, and the algorithm in [2] solves a problem on a linear graph without restrictions on edge capacities in time $O(n^5)$.

In the case of unit demand, the algorithm proposed in the report works in time $O(n^3)$. An algorithm in [3] works with the same time complexity, but in the case of a linear graph and the absence of constraints on the edge capacities.

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[1]The authors are supported by the Russian Foundation for Basic Research grants 16-07-00168 and 15-01-00976.

On a Polynomial Algorithm for the Resource Constrained Multi-Project Scheduling Problem on Random Instances

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We consider a particular case of Resource Constrained Project Scheduling Problem (RCPSP) with single resource of R units and precedence constraints set by oriented acyclic graph G consisting of m connectivity components (that play the role of several independent sub-projects of a certain big project).

An approach of solving this problem is based on two ideas. First, one can note that any feasible solutions of the strip packing problem can serve as feasible solutions of RCPSP with single resource in case of absence of any precedence relations. Thus, approximation algorithms for strip packing problem can be applied to this particular case of RCPSP.

Second, due to acyclic nature of each sub-project component, we can enumerate jobs in each component in the way agreed with the precedence relations; i.e., a for any precedence relation, a predecessor has less number then the successor. Then, we can schedule the set of jobs with number 1 in each project ("layer 1 jobs"), then the set of jobs with number 2 ("layer 2"), using the completion time of previous set a the starting time of current set, and continue in the similar way for all the other layers.

Random instances are generated the following way: each of n jobs is independently getting duration and resource consumption according to some discrete distribution given by the $R \times L$ matrix (p_{rl}). Then, each job is assigned to some of m sub-projects and acyclic set of edges is added to each component.

Using based on results from [1] we prove the following theorem.

Theorem 1. *Assume that each subproject contains k jobs, i.e., $n = mk$. Then, in cases*

- *when (p_{lr}) is L-asymmetric and $k^2W = o(\frac{m}{\ln}m)$*
- *when (p_{lr}) is L-regular and $\frac{k^2W}{\beta_p} = o(\frac{m}{\ln}m)$, where $\beta_p = \frac{1}{WH}\sum_{r=1}^{R}\sum_{l=1}^{L} rlp_{rl}$*

the multi-project RCPSP with single resource is solved asymptotically optimally in polynomial time.

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[1] This work was supported by grants 16-31-00389,16-07-0168 and 15-01-00976 from the Russian Foundation for Basic Research.

An Approximation Algorithm for the Euclidean Maximum Connected k-Factor

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Keywords: connected factor, asymptotically optimal algorithm, Euclidean space.

Given an n-vertex undirected complete graph $G = (V, E)$ without self loops, a weight function $w : E \to \mathbb{R}_+$ and a positive integer $k \geq 2$, the problem is to find a spanning connected k-regular subgraph (connected k-factor) in G of maximum or minimum total weight. This problem is closely related to the network design, where connectivity and degree requirements are common. It is known that the problem is polynomially solvable if there is no requirement for the subgraph to be connected, and NP-hard otherwise. Note that the connected 2-factor problem is the Traveling Salesman Problem.

For the minimum metric connected k-factor problem there is a polynomial approximation algorithm with constant approximation ratio [2].

As long as the connected k-factor problem is a natural generalization of the TSP, and the maximum TSP is in some ways easier than the minimum TSP, the case of the maximum connected k-factor is also of interest.

Paper [1] provides an approximation algorithm for the maximum connected k-factor problem with the relative error $\varepsilon = O\left(1/k^2\right)$ and with $O(kn^3)$ running-time. We note that with more accurate calculations the relative error can be improved to $\varepsilon = O\left(1/k^3\right)$. It is clear that this algorithm is asymptotically optimal for large k that tends to infinity as n grows.

Here we concentrate on the Euclidean maximum connected k-factor problem where the weight of an edge is the Euclidean distance between its endpoints. For this problem in the case of constant or small $k = o(n)$ we present an asymptotically optimal approximation algorithm, which runs in $O(n^3)$ time.

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[1]The authors are supported by the Russian Foundation for Basic Research grants 16-31-00389 and 15-01-00976.

A 5/6-approximation algorithm for the pseudo-metric TSP-max in an incomplete graph

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A metric TSP is a well-known variant of the Traveling Salesman Problem where one need to find a Hamiltonian cycle of an extremal weight in a complete weighted graph whose weight function satisfies the triangle inequality. In 1985, Kostochka and Serdyukov [1] proposed an elegant polynomial 5/6-approximation algorithm for the maximization version of metric TSP (abbreviated as metric TSP-max) based on a delicate rearranging the edge set of a maximum-weight cycle cover of a graph. Since then the approximation ratio in [1] was improved several times by various authors. The best known result is a polynomial 7/8-approximation algorithm for the metric TSP-max by Kowalik and Mucha [2].

The purpose of this paper is to extend the 5/6-approximation algorithm of Kostochka and Serdyukov to the case of pseudo-metric TSP-max where the input graph is incomplete but its minimum degree is sufficiently large and the triangle inequality holds for any triangle of the graph. Observe that the weight function of such a graph is not necessarily extendible to the metric weight function of a complete graph. So our pseudo-metric setting of TSP-max is more general when the modification of metric TSP-max where some edges (of a complete graph) are removed (or unavailable). However, our pseudo-metric TSP-max includes such a modification of the metric TSP-max as an important subcase.

As an easy application of our Algorithm, we produce a simple 5/6-approximation algorithm for the maximization version of the metric m-Peripatetic Salesman Problem (metric m-PSP-max). In the metric m-PSP-max one need to find m edge disjoint Hamiltonian cycles of the maximum total weight in a complete weighted graph with triangle inequality. Recently, such a polynomial 5/6-approximation algorithm for the metric m-PSP-max was presented in [3]. However, its description and analysis in [3] are quite complicated.

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[1]The research is supported by RFBR (projects 15-01-00976 and 15-01-05867).

On $(1, l)$-coloring of incidentors of some classes of graphs

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An *incidentor* in a directed loopless multigraph $G = (V, E)$ is an ordered pair (v, e) where $v \in V, e \in E$ and the arc e is incident with the vertex v. It is convenient to treat the incidentor (v, e) as a half of the arc e adjoining to the vertex v. Each arc $e = uv$ has two incidentors: the *initial* one (u, e) and the final one (v, e). Two incidentors adjoining the same vertex are called *adjacent*. An incidentor coloring is an arbtrary function $f : I \longrightarrow Z_+$, where Z_+ is the set of positive integers (colors). The incidentor coloring is called (k, l)-coloring if: 1) all adjacent incidentors have different colors; and 2) for every arc, the difference between the colors of its final and initial incidentors is in $[k, l]$. The minimum number of colors for which such coloring is possible is denoted by $\chi_{k,l}(G)$.

The notion of incidentor (k, l)-coloring was introduced in [1]. Some bounds on $\chi_{k,l}(G)$ were proved in [2, 3, 4]. In particular, it was proved in [2] that for every graph G of maximum degree Δ and $l \geq \lceil \Delta/2 \rceil$ the bound $\chi_{k,l}(G) \leq \Delta + k$ holds. In this paper we prove the same bound for $k = 1$ and $l = \lceil \Delta/2 \rceil - 1$.

The $(1, 1)$-coloring of incidentors is particulary interesting since the only series of graphs G having $\chi_{k,l}(G) > \Delta + k$ was constructed in [2] for $k = l = 1$ and odd Δ. Moreover, all these graphs had no perfect matching. The author conjecture that evrey graph G with a perfect matching satisfies the bound $\chi_{1,1}(G) \leq \Delta + 1$ and prove this fact for the class of prisms.

Acknowledgments. This work was financially supported by Russian Foundation for Basic Research (prolects 15-01-00976 and 17-01-00170).

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A Combined Method for the Resource Constrained Project Scheduling Problems

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We consider the resource constrained project scheduling problem with precedence and resource constraints (RCPSP). We are given a set of activities $N = \{1, ..., n\}$. The partial order on the set of activities is given by directed acyclic graph $G = (N, A)$. Each activity $j \in N$ is characterized by its deterministic processing time $p_j \in Z^+$ and the resource requirement r_{jk} in resource type k in each unit of time. For each constrained resource type k are known its capacity R^k during the each unit time slot of the planning horizon $\hat{T}$. All resources are renewable. Activities preemptions are not allowed. The objective is to compute the schedule $S = \{s_j\}$ that meets all resource and precedence constraints and minimizes the makespan $C_{\max}(S)$.

The considered problem is NP-hard. We propose to use a combined method to solve the problem. It is based on algorithms of branch and bound and metaheuristics. For the successful use of the branch and bound method the lower bound is very important. We have considered two variants for calculating the lower bound. We propose as the lower bound to use the solution of the relaxed problem where all constrained resources being accumulative. To solve this problem we use a polynomial exact algorithm [1]. As another lower bound, we consider the problem $P|prec, p_j = 1|C_{max}$. In this problem all activities have the unit processing time. V.V. Servah [2] developed a method for solving this problem, based on the dynamic programming idea, the algorithm has an exponential time complexity. At each step of the branch and bound algorithm we apply metaheuristics. We present results of numerical experiments illustrating quality of proposed algorithm. The test instances were used from the library of test problems PSPLIB.

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[1]The authors are supported by the Russian Foundation for Basic Research (project no. 16-07-00829)

Approximation Algorithms for Scheduling Problem with Release Times and Delivery Times

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The problem of minimizing the maximum delivery times while scheduling tasks to parallel identical processors is a classical combinatorial optimization problem. In the notation of Graham *et al.* this problem is denoted by $P|r_j, q_j|C_{max}$ and the special case with one machine is denoted $1|r_j, q_j|C_{max}$.

The goal of this paper is to propose approximate IIT (inserted idle time) [1] algorithms for this scheduling problems.

We consider a system of tasks $U = \{u_1 u_2, \ldots, u_n\}$, which is performed on parallel identical processors. Each task u_i must be processed without interruption for $t(u_i)$ time units on any machine, which can process at most one task at time. Each task u_i has a release time $r(u_i)$, when the task is ready for processing and delivery time $q(u_i)$, its delivery begins immediately after processing has been completed.

A schedule for a task set U is the mapping of each task $u_i \in U$ to a start time $\tau(u_i)$. The task u_i has been delivered at time $L(u_i) = \tau(u_i) + t(u_i) + q(u_i)$. Our objective function is to minimize, over all possible schedules, the maximum delivery time

$$C_{\max} = \max\{\tau(u_i) + t(u_i) + q(u_i) | u_i \in U\}.$$

The stated problem is equivalent to that with release times, due dates instead of delivery times and a maximum lateness criterion which is denoted as $P|r_j|L_{max}$.

We propose 2-approximate inserted idle time algorithms for $P|r_j, q_j|C_{max}$ problem. For $1|r_j, q_j|C_{max}$ we propose a new 3/2- approximate algorithm J/IIT, which runs in $O(n \log n)$ time. The algorithm combines the extended Jackson's rule with algorithm, named EDD/IIT (earliest due date/ inserted idle time).

To compare the effectiveness of proposed algorithms we tested random generated problems of up to 500 tasks. The algorithm J/IIT produced a better solution than algorithm EDD in 71 percent of cases. The solution generated with J/IIT algorithm are on average only 8,3 percent away from the optimal value and this deviation is never more than 12 %.

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On minimizing supermodular set functions on matroids

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Let I be a finite set. A *matroid* on I is a pair $M = (I, \mathcal{A})$, where $\mathcal{A} \subseteq 2^I$ is a family of subsets of I satisfying the following two axioms:
$(A1)$ $(A \in \mathcal{A},\ A' \subseteq A) \Rightarrow A' \in \mathcal{A}$;
$(A2)$ $(A, A' \in \mathcal{A},\ |A| = |A'| + 1) \Rightarrow \exists\ a \in A \setminus A'\ : A' \cup \{a\} \in \mathcal{A}$.
The sets $A \in \mathcal{A}$ are called *independent sets* of M.

A set function $f : 2^I \to \mathbf{R}_+$ is called *supermodular*, if for all $A, B \subseteq I$

$$f(A \cup B) + f(A \cap B) \geq f(A) + f(B).$$

We consider the combinatorial optimization problem:

$$\min\{f(X) : X \in \mathcal{B}\}, \tag{1}$$

where $f : 2^I \to \mathbf{R}_+$ is a nondecreasing supermodular set function, $f(\emptyset) = 0$, and $\mathcal{B}$ is the family of all maximal independent sets (bases) of a matroid $M = (I, \mathcal{A})$.

The well-known NP-hard minimization p-median problem [1] can be reduced to this problem.

We present a performance guarantee of the approximation greedy algorithm for problem (1) using the notion of curvature of the objective function $f(X)$. As a corollary we obtain a bound on worst-case behaviour of the greedy algorithm for the general minimization p-median problem that improves and complements the known bounds [2, 3].

Acknowledgments. The research of the first author was supported by RSF grant 15-11-10009.

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Integer Programming Methods to Polyomino Tiling Problem

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Abstract. This paper presents the approach based on integer programming to the problem of polyomino tiling. Two cases are shown: tiling with L-shaped trominoes and tiling with L-shaped tetrominoes. The IP mathematical model and tiling algorithm is described. This problem can be applied to the phased array design where polyomino-shaped subarrays are used to avoid the regularity of antenna structure [1]. Simulations of antenna perfomance show good suppression of sidelobes while the structure fullness is close to maximum.
Keywords: polyomino, integer programming, phased array.

We consider the tiling of finite, rectangular region with given polyominoes, without any restriction on their number. Each polyomino can be rotated by 90 degrees and mirror-flipped. So, there is an $N \times N$ element region and infinite number of polyominoes. The problem is to find an optimized layout of polyomino considering two following requirements: to minimize the number of empty spaces and to maximize irregularity i.e. eliminate periodicity of polyominoes layout [2]. Let an $N \times N$ element structure be represented as the set of binary variables $z_{ij} = \{0, 1\}$ where $i = 1, .., N$ and $j = 1, .., N$. Let $z_{ij} = 1$ if it contains the center of polyomino and $z_{ij} = 0$, otherwise. The objective function is to maximize the sum of all variables z_{ij}.

Presented mathematical model and tiling algorithm were implemented in program using Python and IBM ILOG CPLEX solver. Obtained tilings were used to simulate antenna performance. For the case of L-tromino the best peak sidelobe level is -29.15 dB for r = 1.3 with the structure fullness of 99.9%. For the case of L-tetromino the best peak SLL is -25.69 dB for r = 1.3 with the structure fullness of 100%.

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New heuristic for searching BPP and DBPP instances with large proper gap

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Abstract. We consider the bin packing problem and the dual bin packing problem, with the aim of maximizing the best known (proper) gaps. Using new heuristics, we improved the best known proper gap from 1.0625 to 1.0925. The best known dual gap is raised from 1.0476 to 1.1775. The proper dual gap is improved to 1.1282.
Keywords: bin packing problem; dual bin packing problem; proper relaxation; gap; proper gap; dual gap; proper dual gap

We consider the bin packing problem, the dual bin packing problem and its proper and continous relaxations based on the formulation by Gilmore and Gomory [GG1]. The following inequality shows relations between objective values of the considered problems:

$$\overline{z}(E) \leq \overline{z}_P(E) \leq \overline{z}_C(E) \leq z_C(E) \leq z_P(E) \leq z(E).$$

Also there is a series of gaps: the gap $\Delta(E) = z(E) - z_C(E)$, the proper gap $\Delta_p(E) = z(E) - z_P(E)$, the dual gap $\overline{\Delta}(E) = \overline{z}_C(E) - \overline{z}(E)$ and the proper dual gap $\overline{\Delta}_p(E) = \overline{z}_P(E) - \overline{z}(E)$. The main open problem here is whether there is an instance with the gap or the dual gap greater than 2.

The search is based on the method described by Kartak, Ripatti, Scheithauer and Kurz [KRSK1] for the bin packing problem. This method is a branch and bound search with special cuts. We adapt it for the dual bin packing problem.

The heuristic is based on the following observation. The set of patterns for n items under the domination relation (introduced in [KRSK1]) is isomorphic to the well-known poset $M(n)$ introduced by Stanley [St1]. We experimented with the rank function of poset $M(n)$ and developed a new cut that allowed us to get the results presented in the abstract.

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The equivalent transformation of the d-dimensional Orthogonal Packing Problem

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Abstract. We consider the d-dimensional Orthogonal Packing Problem and present an effective algorithm for building equivalent OPP-d instances with certain properties. We apply the developed toolset for building equivalent instances with the reduced number of the raster points.
Keywords: orthogonal packing problem; bin packing problem; knapsack problem; conservative scale; equivalence of instances; raster points; raster model.

We consider the well-known d-dimensional Orthogonal Packing Problem (OPP-d), which can be formulated as follows. A set of d-dimensional items (rectangular boxes) needs to be packed into a fixed container. The input data describe the dimensions of the container $W_k \in \mathbb{R}_+$, $1 \leq k \leq d$, and the dimensions of the n items $w_i^k \in \mathbb{R}_+$, $1 \leq k \leq d$ for each item $1 \leq i \leq n$. We ask whether all boxes can be orthogonally packed into the container without rotations.

Using the toolset of conservative scales introduced by Feteke and Schepers [FS1] we are able to change the items' sizes in the initial instance to obtain an equivalent instance with the same solution. We present an effective algorithm for building equivalent instances with certain properties.

We also consider the so-called raster model for OPP-d introduced by Belov, Kartak, Rohling and Scheithauer [BKRS1]. It is a 0/1 ILP model in which number of variables and constraints linearly depends on the total number of raster points over all dimensions. Using our algorithm we construct equivalent instances with reduced number of raster points. We also present an algorithm to find a lower bound on the minimum possible number of raster points over all equivalent instances. For some instances, it proves that it is impossible to reduce the number of raster points. Numerical results are presented.

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A Randomized Algorithm for Two-Cluster Partition of a Sequence

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In the paper we consider the following strongly NP-hard [1]

Problem. *Given* a sequence $\mathcal{Y} = (y_1, \ldots, y_N)$ of points from $\mathbb{R}^q$, and some positive integer numbers $T_{\min}$, $T_{\max}$, and M. *Find* a subset $\mathcal{M} = \{n_1, \ldots, n_M\}$ of $\mathcal{N} = \{1, \ldots, N\}$ such that

$$\sum_{j\in\mathcal{M}} \|y_j - \overline{y}(\mathcal{M})\|^2 + \sum_{i\in\mathcal{N}\setminus\mathcal{M}} \|y_i\|^2 \to \min,$$

where $\overline{y}(\mathcal{M}) = \frac{1}{|\mathcal{M}|}\sum_{i\in\mathcal{M}} y_i$, under constraints

$$T_{\min} \le n_m - n_{m-1} \le T_{\max} \le N,\ m = 2, \ldots, M,$$

on the elements of $(n_1, \ldots, n_M)$.

This problem is important, for example, in time series analysis, data mining, machine learning, and noise-proof clusterization of signals.

In this work, we present a randomized algorithm for the problem. Under assumption $M \ge \beta N$, where $\beta \in (0, 1)$ is some constant, and given $\varepsilon > 0$ and $\gamma \in (0, 1)$, the algorithm finds a $(1 + \varepsilon)$-approximate solution of the problem with probability not less than $1 - \gamma$ in $\mathcal{O}(qMN^2)$ time. The conditions are found under which the algorithm finds a $(1+\varepsilon_N)$-approximate solution of the problem with probability not less than $1 - \gamma_N$, where $\varepsilon_N \to 0$ and $\gamma_N \to 0$ as $N \to \infty$, in $\mathcal{O}(qMN^3)$ time.

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An Approximation Scheme for a Weighted 2-Clustering Problem

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We consider the following clustering problem:

Given an N-element set $\mathcal{Y}$ of points from $\mathbb{R}^q$, a positive integer $M \le N$, and real numbers (weights) $\omega_1 > 0$ and $\omega_2 \ge 0$. *Find* a partition of $\mathcal{Y}$ into two clusters $\mathcal{C}$ and $\mathcal{Y}\setminus\mathcal{C}$ minimizing the value of

$$\omega_1 \sum_{y\in\mathcal{C}} \|y - \overline{y}(\mathcal{C})\|^2 + \omega_2 \sum_{y\in\mathcal{Y}\setminus\mathcal{C}} \|y\|^2,$$

where $|\mathcal{C}| = M$ and $\overline{y}(\mathcal{C}) = \frac{1}{|\mathcal{C}|}\sum_{y\in\mathcal{C}} y$ is the centroid of $\mathcal{C}$.

We present an algorithm that allows to find a $(1+\varepsilon)$-approximate solution of the problem in $\mathcal{O}\Big(\sqrt{q}N^2\big(\frac{\pi e}{2}\big)^{q/2}\big(\frac{1}{\sqrt{\varepsilon}}+2\big)^q\Big)$ time for any $\varepsilon \in (0,1)$. The algorithm implements a FPTAS in the case of fixed space dimension and remains polynomial for instances of dimension $\mathcal{O}(\log n)$.

Earlier, in [1, 2, 3], for the strongly NP-hard cases when (1) $\omega_1 = 1$ and $\omega_2 = 0$, (2) $\omega_1 = \omega_2 = 1$ and (3) $\omega_1 = |\mathcal{C}|$ and $\omega_2 = N - |\mathcal{C}|$, the approximation schemes with running time $\mathcal{O}\Big(qN^2\big(\sqrt{\frac{2q}{\varepsilon}}+1\big)^q\Big)$ were proposed.

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Approximation schemes for the Euclidean CVRP with non-uniform demand and time windows

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Capacitated Vehicle Routing Problem (CVRP) is the well-known combinatorial optimization problem is introduced by G. Datzig and J. Ramser in their seminal paper [1]. As known (see, e.g. [2]), this problem is NP-hard even in finite dimensional Euclidean spaces. Although the problem is hardly approxmable in general, its geometric settings can be approximated rather well. Most of the known results in this field date back to the famous papers by M. Haimovich and A. Rinnoy Kan [3] and S. Arora [4]. Above these results, the most recent seem to be the quasipolinomial time approximation scheme proposed in [5] for the Euclidean plane and extended in [6] to the case of finite number of non-intersecting time windows, and (to the best of our knowledge) the first polynomial time approximation scheme introduced in [7] for the CVRP formulated in d-dimensional Euclidean space for an arbitrary $d > 1$.

In this paper, we propose an extension of the results of [6] to the case of d-dimensional Euclidean spaces and non-uniform customer demand following the general framework developed in [7].

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[3] Supported by RSCF grant 14-11-00109

Efficient optimal algorithm for the Quasipyramidal GTSP[1]

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The Traveling Salesman Problem (TSP) is one of the famous NP-hard combinatorial optimization problems. Although TSP is intractable and hardly approximable in its general case, there are restricted settings of the problem, which can be solved to optimality in polynomial time. For instance, it is known [1] that, for any non-negative weight function w, in weighted complete (di)graph $G = (\mathbb{N}_n, E, w)$, an optimal *pyramidal* Hamiltonian cycle, i.e. the closed tour

$$1, i_1, \ldots, i_{r-1}, i_r = n = j_{n-r}, j_{n-r-1}, \ldots, j_1, 1, \tag{1}$$

for which $i_s < i_{s+1}$ and $j_t < j_{t+1}$ for each $0 \leq s < r-1$ and $0 \leq t < n-r-1$, can be found in time of $O(n^2)$.

In [2, 3] this result is extended to the special cases of quasipyramidal tours (called by the authors as tours with *step-backs* and *jump-backs*). Actually, for some fixed l, tour (1) is called *l-quasipyramidal* if $i_p \leq i_q + l$ and $j_u \leq j_v + l$ for any $1 \leq p < q \leq r$ and $1 \leq u < v \leq n - r$. As it is proven in [3], an optimal l-quasipyramidal tour can be found in time of $O(8^l n^2)$.

In this paper we propose an extension of the aforementioned results to the case of the Generalized Traveling Salesman Problem, where a partial order on the vertex set V of a given graph $G = (V, E, w)$ is induced by the linear ordering of the clusters $V_1, \ldots, V_k$. Along with presenting the appropriate polynomial time algorithms finding optimal pyramidal and quasipyramidal tours in this case, we propose novel extensions of the well-known Demidenko and van der Veen sufficient conditions providing existence of an optimal GTSP solution in the subclass of (quasi)pyramidal tours.

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[1] Supported by RSCF grant 14-11-00109.

Minimizing machine assignment costs over Δ-approximate solutions of $P||C_{\max}$

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Bi-criteria lexicographical minimization problems with the makespan as the primary objective and the total machine assignment costs as the secondary objective have been recently introduced to the scheduling research, and polynomial time $(\Delta, 1)$-approximation algorithms have been suggested for their solution [1]. We study a problem of minimizing the total machine assignment cost over the Δ-approximate solutions of the makespan minimization problem. We prove that this new problem is strongly NP-hard and pseudo-polynomially non-approximable in general. A polynomial time approximation algorithm with a guaranteed approximation ratio is presented for a special case where the ratio between the maximal and minimal costs associated with the machines is bounded. An $O(mn^{2k})$ time dynamic programming algorithm is presented for another special case in which the number k of distinct job processing times is fixed.

Acknowledgments. The research of the first author is supported by the Russian Science Foundation grant 15-11-10009.

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Local search for multicriteria single machine scheduling with setups

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In this paper we consider a single-machine scheduling problem with setup times. We are given a set J of n jobs. Each job j consists of a number j_k of operations. Each operation correspond to manufacture of products of certain type. Each operation $o_i \in j$ has processing time p_i, weight v_i and a setup time s_{ik} which is incurred when operation o_k immediately follows operation o_i. A due date d_j is specified for each job j. The machine is continuously available through the planning period and can process at most one operation at a time. Once an operation is started it must be completed without interruption.

We denote the completion time of operation o_i by C_i. The tardiness L_j of job j is defined as $L_j = \max\{0, \max_{o_i \in j}(C_i - d_j)\}$, that is, the positive time difference between the due date of job j and the completion time of the last operation of job j. The earliness e_i of operation o_i of job j is defined as $e_i = \max\{0, (d_j + L_j - C_i)v_i\}$. The value e_i determines the storage costs of products of type i.

We want to find a schedule in which operations are to be completed as close to their due dates as possible and at the same time to minimize the makespan. Thus, in our problem , it is required to find a sequence of operations which delivers the minimum value to the function

$$F(\pi) = \sum_{j=1}^{n}(\alpha_j L_j(\pi) + \sum_{o_i \in j} \beta_i e_i) + \gamma C_{max}.$$

We present a local search algorithm to solve this problem. Computational results and some open questions are discussed.

Acknowledgements. This research is supported by the Russian Science Foundation grant 15-11-10009.

Variable Neighborhood Search Algorithms for Competitive p-Median Facility Location Problem

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The facility location problems are one of the most studied areas of operations research. The essence of these problems is to locate facilities in some points and assign the consumers to them for service. The goal is to minimize the total cost. There are a number of models that take into account the market competition. Aboolian R. et al. [1] formulated a facility location and design problem (CFLDP) in which the facilities compete for the serviced market share. Such share is elastic and depends on where and which kind of facility is located. In this paper, a new formulation of the problem is given, in which the restriction on the quantity of open facilities is added. The number of new facilities is fixed and is equal to p, so competitive p-median facility location and design problem is derived (CPFLDP).

Because of the non-linear objective function of corresponding mathematical models, it is particularly hard to find the optimal solutions in large-dimensional cases. As our computational experiment showed, fixing the number of opened facilities in CFLDP significantly increases the runtime of commercial software. Therefore, the development of approximate methods of solution is particularly is important. Previously, a number of approximation algorithms were proposed for CFLDP, including a variable neighborhood search method (VNS) [1, 2]. In this paper, we develop VNS approach for CPFLDP. Variants of VNS algorithms for the specified problem are offered and a new kind of neighborhood is proposed. Series of testing instances are conducted, the obtained results are discussed.

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Binary Cuts Algorithm for Mixed Integer Programming Problems

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A common feature shared by most implementations of mixed integer linear programming (milp) problems is the NP class membership, large dimensions, and complexity of constraint structures. The focus of this paper is on a method of solving milp problems, which is based on binary cuts (BCs) [1]. One of its algorithms, a hybrid algorithm based on binary cuts and branches (binary cut-and branch algorithm (BCBA)), which combines the idea of the branch-and-bound method with the construction of cutting planes, is extended to milp problems. The results of this extension are discussed. The following problem is considered:

$$\gamma(x) = {c^1}^T x + {c^2}^T y + const \to \max \tag{1}$$

$$A^1 x + A^2 y \leq b, \bar{0} \leq x \leq \bar{1}, y \geq \bar{\bar{0}}, \tag{2}$$

$$x \in I_2^{n^1}, \tag{3}$$

which is a milp problem with Boolean variables and continuous variables. Suppose x^0, y^0 is the solution of the relaxed problem (1)-(2); $[\cdot]$ is the integer part of number; and $\beta_0 = \widehat{\alpha}^T x^0$, where $\widehat{\alpha}_j \in \{0,1\}$, $j = \overline{1, n^1}$. Then a BC for problem (1)-(2) is defined as an inequality of the form $\widehat{\alpha}^T x \leq \widehat{\beta}_0, \widehat{\alpha}_j \in \{0,1\}$, $j = \overline{1, n^1}, \widehat{\beta}_0 = [\beta_0]$, $\beta_0 = \widehat{\alpha}^T x^0$

The BC-generating inequality is $\varsigma^T x \leqslant \phi_0, \phi_0 = \varsigma^T x^0$, under the conditions $\varsigma_j = \sum_{i \in I^B} \lambda_i a_{ij}$, $\lambda_i \geqslant 0$, where $a_{ij}, i \in I^B$ are the coefficients of the basis matrix and λ_i are the weights of the basis constraints. Each $\widehat{\alpha}^j$ is set in correspondence with the value $cs(\widehat{\alpha}^j) = \frac{\varsigma^T \widehat{\alpha}^j}{|\varsigma|_2 \left|\widehat{\alpha}^j\right|_2}, j = \overline{1, n}^1$. $cs(\widehat{\alpha}^j)$ defines how close each cut with the coefficients $\widehat{\alpha}^j$ is to the generating inequality (proximity measure).

Another important feature of BCs is their radicality measure r, which is defined as the number of unit hypercube vertices cut off by the BC, assuming that the cut is valid. For $\widehat{a}^T x \leq b$, $b \in I_1^k, x \in I_{n^1}^2, \widehat{a} \in I_{n^1}^2$, $k = \sum_{j=1}^{n^1} \widehat{a}_j, 1 \leq k \leq n^1$ and ${}_k^l = \frac{k!}{l!(k-l)!}$, we define: $r_k^b = 2^{n^1 - k} \sum_{l=b+1}^{k} {}_k^l \to \max$.

Practical applications identified several classes of milp-reducible problems differing in the efficiency of the two measures in the BCBA. In problems allowing the construction of valid BCs, the use of the measure $cs(\widehat{\alpha}^j)$ allows the synthesis of valid cuts for on average 75percen of the algorithm steps, and the BCBA shows in experiments a speed that is statistically close to that of the polynomial algorithm [1]. However, there are classes of problems for which there are no valid BCs. Examples include makespan scheduling problems for parallel machines with delays in job entering and related problems [2]. In these cases, the radicality measure r proved to be much more efficient, leading, in some cases, to an increase in the speed of the BCBA computations by many orders of magnitude. The computational experiments also revealed the potential of using different BC construction strategies. The algorithm allows the use of a large menu of strategies with BCs that are best tailored to the structure of constraints (2).

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Minimizing Resource Cost in Project Scheduling Problem with Accumulative Resources

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We consider a project with a set of jobs $J = \{1, \dots, n\}$. Precedence constraints are given by a directed acyclic graph $G = (J, E)$, where vertices correspond to jobs and an arc (i, j) belongs to E if and only if job i is a direct predecessor of job j. All jobs have to be completed before the due date T of the project. To be processed, the jobs require accumulative resources that are purchased. Let K denote the set of resources. While being processed, job $j \in J$ requires q_{kj} units of resource $k \in K$ during every period of its non-preemptable duration p_j. The purchase cost of a resource depends on used amount of it per time period. If the amount of the resource $k \in K$ is less than V_k^{norm}, then the cost of resource k per unit equals c_k^{norm}. When this level is exceeded, the new price c_k^{over} is set. Both cases $c_k^{over} > c_k^{norm}$ and $c_k^{over} < c_k^{norm}$ are considered. The goal is to find a feasible schedule and a planning of purchasing and using of resources so that the total resource cost is minimized. This problem with renewable resources is known as Resource Availability Cost Problem (see for ex. [1], [2]).

For unlimited storage, when each resource can be stored in any amount, both considered cases can be solved in polynomial time. However limited storage is more interesting from the practical point of view. In this case, the surplus of a resource k, that is, the resource amount which may be spent later should not exceed the value V_k^{cont}, $k \in K$, in each period of time.

In this paper, we prove NP-hardness and propose integer linear programming models for the considered cases of the problem with limited storage. Dynamic programming algorithms and some heuristics are also developed.

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Heuristic Algorithms for Multi-Channel Convergecasting in Wireless Networks

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Keywords: wireless networks, energy efficiency, information desimination, multi-channel convergecasting

Minimization of the data aggregation time in the communication networks is one of the most important issues related to the prolonging the network lifetime. We consider a problem of construction a minimum length convergecasting schedule in multi-channel wireless networks in a case of unbounded number of channels. This problem may occur in the different wireless networks where the number of the frequencies is rather large, and therefore only the conflicts between the children of the same parent in the aggregation tree are taken into account.

This problem is equivalent to the well-known NP-hard telephone broadcasting problem, formulated in [1]. We propose new heuristic algorithms based on the tree generation techniques, developed for the approximate solution of Min-Power Symmetric Connectivity Problem [2, 3]. We have performed an extensive simulation, which demonstrated the advantage of our algorithms compared to the best of the previous approaches from [4, 5].

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New Guaranteed Errors Greedy Algorithms in Convex Discrete Optimization Problems in terms of Steepness Utility Functions

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Abstract. As is known (see, [1, 2]), a guaranteed error estimate for the gradient algorithm as applied to some discrete optimization problems can be expressed in terms of the steepness of the utility functions. We obtain improved guaranteed estimates for accuracy of the gradient algorithm.

We consider the following convex discrete optimization problem A: find

$$\max\{f(x) : x = (x_1, ..., x_n) \in P \subseteq Z_+^n\},$$

where $f(x) \in \Re_\rho(Z_+^n)$[1], $f(x)$ is a nondecreasing function on the set P, P is an ordinal-convex set [3]. Let $x^g = (x_1^g, ..., x_n^g)$ be the gradient solution (the gradient maximum of the function $f(x)$-on the set P) of the problem A [1, 3].

Let c be steepness of the function $f(x)$ on the set P [1,2].

Theorem. Let $f(x) \in \Re_\rho(Z_+^n)$ be a nondecreasing on the set P function.Then the global maximum x^* and the gradient maximum x^g of the function $f(x)$ on the set $P \subseteq Z_+^n$ satisfy the relation

$$\frac{f(x^*) - f(x^g)}{f(x^*) - f(0)} \leq \left(\quad 1 - \frac{1}{1 + (1 - c)(h - 1)} \quad \right)^r - B,$$

where $h = \max\{h(x) = \sum_{i=1}^{n} x_i : x = (x_1, ..., x_n) \in P\}, 0 \leq B \leq 1, r = \min\{h(x) - 1 : x \in Z_+^n \backslash P\}$.

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Faster approach for exhaustive search over graphs with certain properties

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Abstract. We present new techiques for exhaustive search over graphs with certain properties. We apply them to the cubic version of the Erdős-Gyárfás conjecture and computationally show that the smallest cubic graphs without 4,8,16 cycles have 78 vertices.
Keywords: Erdős-Gyárfás conjecture; cubic graphs; exhaustive search.

The cubic version of the Erdős-Gyárfás conjecture asserts that every cubic graph (regular graphs of degree 3) contains a cycle whose length is a power of 2.

Using computer search Royle and Markström showed that any example must have at least 17 vertices, and a cubic example must have at least 30 vertices [Ma1]. They also found 4 minimal cubic graphs of order 24 without 4,8 cycles (but having cycle of length 16). Exoo constructed cubic graph of order 78 without 4,8,16 cycles [Ex1] and asked whether there are smaller examples here.

We show that the graph found by Exoo is indeed the smallest example of a cubic graph without 4,8,16 cycles, and also present 5 other such graphs.

Our approach is based on a framework developed by McKay for isomorphism-free exhaustive generation of combinatorial objects [Mc1] and fast generation of cubic graphs by Brinkmann [Br1]. We implemented a program similar to the program `minibaum` written by Brinkmann with a few special modifications.

The main idea of our approach is using a probabilistic checking of every node of the search tree. This method increases the number of nodes we have to consider, but significantly reduces the average time for checking every particular node. Thus, it reduces the total computing time. Our program turned out about 300 times faster than the `minibaum` program.

Our techniques can be applied to other graph problems too.

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Optimization Model for the Harbor Scrap-Metal Logistic

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We study a new optimization model to minimize the total operational cost of a logistic company on a given time horizon. The company has some local providers who supply it by scrap-metal materials of different qualities. The materials are manufactured into the high quality product and exported to abroad by heterogeneous fleet of ships. Company can send at most one ship per day. The total demand for the product is known in advance according to a contract. The company has to pay to providers according to piecewise linear prices, transportation cost to deliver all materials to depot in harbor by vehicles (own or rented), keeping cost in depot, manufacturing cost, the shipping cost, and payment for international declarations. The goal is to find the best strategy for the company to minimize the total cost to perform the contract. We present a mixed integer linear programming formulation for the model. The CPLEX software can find optimal or near optimal solutions for small horizon only. Thus, we design a mat-heuristic and use CPLEX solver for moving time intervals. We conduct computational experiments on real test instances and discuss so-called tail's effect.

Acknowledgments. This research was partially supported by RFBR grant 16-07-00319.

The Longest Vector Sum Problem: Complexity and Algorithms

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In the *longest vector sum problem* (LVS), we are given a set X of n vectors in a normed space $(\mathbb{R}^d, \|.\|)$, the goal is to find a subset $S \subseteq X$ maximizing the value of $\left\| \sum_{x \in S} x \right\|$. The variation where the subset S is required to have a given cardinality $k \in [1, n]$ will be called the *longest k-vector sum problem* (Lk-VS).

Both of these problems are strongly NP-hard if the Euclidean norm is used [4, 1] and polynomially solvable when d is fixed. The best known algorithm for the general LVS runs in time $O(n^d)$ [3], the Lk-VS can be solved in time $O(n^{4d})$ [2]. In the Euclidean case, both problems admit an FPTAS for any fixed d [1].

Our Contributions. First, we prove that, for any ℓ_p norm, $p \in [1, \infty)$, the LVS and Lk-VS problems are hard to approximate within a factor better than $\min\{\alpha^{1/p}, \sqrt{\alpha}\}$, where α is the inapproximability bound for Max-Cut, $\alpha = 16/17$ (or $\alpha \approx 0.879$ if the Unique Games Conjecture holds).

On the other hand, for an arbitrary norm, we reduce the computational time for both problems. We propose an $O(dn^{d-1} \log n)$-time algorithm for the LVS and an $O(dn^{d+1})$-time algorithm for the Lk-VS. In particular, the two-dimensional LVS problem can be solved in a nearly linear time.

Finally, for any norm, we propose a randomized algorithm which finds $(1-\varepsilon)$-approximate solutions of both problems in time $O(d\,(1+\frac{2}{\varepsilon})^d\, n)$. In particular, we have a linear-time algorithm for any fixed d and ε. The algorithm is polynomial for instances of dimension $O(\log n)$.

Acknowledgments. This research is supported by Russian Science Foundation (project 16-11-10041).

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Scheduling Problems with Controllable Processing Time to minimize Minmax and Quadratic Objectives

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In scheduling with controllable processing times (SCPT) the actual durations of the jobs are not fixed in advance, but have to be chosen from a given interval. For a SCPT model, two types of decisions are required: (i) each job has to be assigned its actual processing time, and (ii) a schedule has to be found that provides a required level of quality.

The jobs of set $N = \{1, 2, \ldots, n\}$ have to be processed either on a single machine M_1 or on parallel machines $M_1, M_2, \ldots, M_m$, where $m \geq 2$. For each job $j \in N$, its processing time $p(j)$ is not given in advance but has to be chosen from a given interval $[l(j), u(j)]$. That selection process is often seen as *compressing* the longest processing time $u(j)$ down to $p(j)$. The value $x(j) = u(j) - p(j)$ is called the *compression amount* of job j.

Each job $j \in N$ is given a *release date* $r(j)$, before which it is not available, and a *deadline* $d(j)$, by which its processing must be completed. In the processing of any job, *preemption* is allowed, so that the processing can be interrupted on any machine at any time and resumed later, possibly on another machine. It is not allowed to process a job on more than one machine at a time, and a machine processes at most one job at a time. A schedule is called *feasible* if the processing of a job $j \in N$ takes place in the time interval $[r(j), d(j)]$.

Each job can be associated with several weights $w_T(j), w_M(j)$ and $w_Q(j)$, which serve as coefficients that define an objective function that depend on the compression amounts. Typical functions include (i) total compression cost $\Phi_\Sigma = \sum_{j \in N} w_T(j)\, x(j)$, (ii) maximum compression cost $\Phi_{\max} = \max\{x(j)/w_M(j)\,|j \in N\}$, and (iii) quadratic compression cost $\Phi_Q = \sum_{j \in N} \left(w_Q(j)\, x(j)^2 + w_T(j)\, x(j)\right)$.

Problems that involve minimization of function Φ_Σ are well-studied within the SCPT area. The function $\Phi_{\max}$ is a popular objective within the body of research on scheduling with imprecise computation. The quadratic function Φ_Q, although obviously relevant, has not received considerable attention so far.

In this paper, we report on the progress in solving the problems that involve minimization of these functions, including a multicriteria environment, either for the lexicographically ordered criteria or in the Pareto sense. For many problems in this range polynomial-time algorithms can be designed.

On a new class of facet inducing inequalities for the correlation clustering problem

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In this paper we continue our polyhedral research for the correlation clustering problem [1]. Let $\mathbf{K}_n = (V, E)$ be a complete unoriented graph without loops and multipe edges. Spanning subgraph $H \subset \mathbf{K}_n$ is called M-graph if each of its connected components is a clique or single-vertex graph. We denote the set of all M-graphs in $\mathbf{K}_n$ through $\mu(V)$. Let $G \subset \mathbf{K}_n$ be some a priori set spanning subgraph. Correlation clustering problem consists in finding M-graph H minimizing the functional $\rho_G(H) = |EG \cup EH| - |EG \cap EH|$ on set $\mu(V)$. We present this problem as the problem of minimizing a linear functional on the convex hull of the incidence vectors of M-graphs. In this paper a new class of inequalities that induce the facets of a polytope of a problem are described, the separation problem for these inequalities are discussed.

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A cutting plane method for the graph approximation problem

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In this paper we continue our polyhedral research for the graph approximation problem [1]. We present this problem as a linear integer program and apply the cutting plane method based on the so-called k-parashuties inequalities. The separation problem for k-parashuties is NP-hard [2]. Thus, we design fast local search algorithm to discover them for arbitrary vertex of polyhedral. Each iteration of our cutting plane method consists in the following. For the current optimum for the linear programming relaxation, we solve the separation problem to generate some k-parashuties inequalities. If we discover them, these inequalities are included into the set of constraints for the graph approximation problem. Otherwise, Gomory's cutting are used.

Computational experiments for graphs with 100 vertexes are presented. The aims of the experiments is to analyze the performance of the cutting plane method and local search algorithm for the separation problem for the k parachutes.

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Genetic fuzzy clustering algorithm with greedy crossing-over procedure

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Results of testing of the electronic components shipped for the space industry are represented by arrays of data vectors of very high dimensionality, up to hundreds of dimensions. One of the most important problems for increasing the quality of the electronic units is detection of the homogeneous production batches of the electronic devices. We consider the problem of fuzzy clustering of data sources applying new genetic algorithm with greedy agglomerative heuristic based on the EM algorithm for precipitation of mixture of Gaussian distributions.

Each solution (individual) in this algorithm is represented by a pair $< D, W >$ of a set of the distributions $D = \{N(\mu_i, \sigma_i^2, i = \overline{1,k})\}$ and a set of weight coefficients of distributions $W = \alpha_i, i = \overline{1,k}$. During the crossing-over procedure, sets of two selected pairs $< D', W' >$ and $< D'', W'' >$ are joined: $D = D' \cup D''$, $W = W' \cup W''$, then the following greedy procedure [1] runs:

1. Run the EM algorithm with $< D, W >$.
2. If $|D| = k$ then stop.
3. For each $i' \in \{\overline{1, |D|}\}$ do: assign the truncated set $D' = D \setminus \{N(\mu_{i'}, \sigma_{i'}^2)\}$, $W' = W \setminus \{\alpha_{i'}\}$. Run one iteration of the EM-algorithm with D^{TM} and W^{TM}. Calculate the objective likehood function L for the received result, store its value in $L_{i^{\text{TM}}}$. Continue loop 3.
4. Find index $i'' = \arg\max_{i'=\overline{1,K}} L_{i'}$. Assign the truncated sets $D = D \setminus \{N(\mu_{i''}, \sigma_{i''}^2)\}$, $W = W \setminus \{\alpha_{i''}\}$. Run the EM-algorithm for D and W. Go to Step 2.

Computational experiments with electronic component testing data and classical datasets for clustering problems show that this new algorithm allows to obtain more precise results in comparison with classical EM algorithm and its modifications. For tests of the IC 1526LE2 (number of data vectors$N = 3987$, dimensionality $d = 206$) average result of likelihood function of new algorithm is 443491, average result of EM-algorithm is 350292.

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Evaluating heuristics via search tree size estimation

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D. A. Knuth has pointed out that the size and the profile of the search tree associated with a given back-track algorithm can reasonably be well estimated by certain random exploration. We propose to use this random exploration technique to estimate the computational costs such as running time in connection with an exhaustive back-track search. Many combinatorial optimization problems resort on exhaustive searches. From reason of practicability it is necessary to combine the search with various heuristic principles. The possibility of estimating the running time of a search without actually carrying out the computation is far reaching and be can be exploited beneficially. Usually there are more than one choice for a particular component of an algorithm. In other words there maybe competing candidates to employ as a part of an algorithm to produce a certain effect. In these cases one can base the choice between the contenders on the estimations of the respective running times.

We will compile a battery methods to overcome a given obstacle or accomplish a certain result in the course of a computation. We do not decide which technique should be used a priory. Rather we estimate the running times of the task under each scenario in running time (on line) and we place our bet on the fastest. We will illustrate the methodology in connection with the k-clique problem as a typical example of a computationally challenging, NP-complete combinatorial optimization problem. Based on the Carraghan–Pardalos algorithm [1] one can choose among several different coloring techniques as heuristics for cutting off the branches of the search tree. Such are the b-fold coloring [5], triangle-free and s-clique free colorings [3] and special edge colorings [4]. Also rearranging the branches can result in reducing the size of the search tree [2].

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A polynomial 3/5-approximation algorithm for the problem of finding three edge disjoint Hamiltonian circuits of the maximum weight in a complete digraph

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The m-Peripatetic Salesman Problem (m-PSP) (introduced by Krarup in 1975) is a natural generalization of the Traveling Salesman Problem (TSP). In m-PSP one need to find m edge disjoint Hamiltonian cycles of minimum or maximum total weight in a complete weighted graph. In this paper we investigate the asymmetric maximization version of the m-Peripatetic Salesman Problem (m-APSP-max). The input of the problem is a complete directed graph G and a non-negative weight function of its edges. The task is to find m edge disjoint Hamiltonian circuits of maximum total weight in G.

It is known that the problems of finding one or more Hamiltonian circuits in a digraph are NP-hard. So the efforts of most researchers are concentrated on finding cases where the problem can be solved in polynomial time and developing polynomial algorithms with guaranteed approximation ratios for such problems.

The best known approximation algorithm for the ATSP-max has the guaranteed ratio 2/3 [1]. Authors of [2] developed an algorithm with the same ratio for the 2-APSP-max and an algorithm on random instances for the m-APSP for which the conditions of its asymptotical exactness were established.

However, for $m > 2$, no deterministic approximation algorithms for the m-APSP-max are constructed. The main result of this paper is a deterministic Algorithm for the 3-APSP-max with approximation ratio 3/5 and cubic running time. Following the ideas in [2], our Algorithm starts with an acyclic edge colouring of a specified regular subgraph in G. Such a colouring allows us to construct three partial tours with sufficiently large weight in G and to extend them to three edge disjoint Hamiltonian circuits which form a desired 3/5-approximate solution of the 3-APSP-max.

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[1]The research is supported by RFBR (project 16-01-00108)

Bin Covering of Subsets

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Abstract. We extend the bin covering problem of numbers [1, 2] to the bin covering problem of subsets and discuss its application in wireless sensor networks. Given a lower bound k on the amount that each bin must be filled, the bin covering problem of numbers aims to distribute a given set of numbers into as many bins as possible. The bin covering problem of numbers is NP-hard, and research has concentrated on polynomial-time approximation algorithms. For bin covering of subsets, we are given a family T of subsets of a ground set U, and the problem is to partition T into as many bins as possible so that each bin contains at least k elements of U. First we propose a greedy heuristic for the bin covering problem of numbers that delivers a $1 + \frac{2}{H_{|S|}}$ approximation in which H_n is the n-th harmonic number. The greedy heuristic is extended to deal with the bin covering problem of subsets. The extended heuristic achieves a $1 + \frac{2H_k}{H_{|T|}}$ approximation. To apply the algorithm to Wireless Sensor Networks (WSN), we consider U as a set of targets to be monitored , and each sensor is denoted by a subset of U consisting of the targets in its sensing range. The Maximum lifetime Target Coverage (MTC) problem [3, 4] in Wireless Sensor Networks (WSN) is to divide the set of sensors into as many groups as possible so that each group can cover all the targets. Since the groups of sensors take turns monitoring all the targets, more groups means longer lifetime of the WSN. That is, the problem can be reduced to the bin covering problem of subsets with $k = |U|$.

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Weber Problem on Line with Forbidden Gaps[1]

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The Weber problem on a line with forbidden gaps is defined as follows. It is necessary to locate facilities $X_1, \dots, X_n$, which are the segments, on a straight–line segment of length LS containing some fixed segments (forbidden gaps) $F_1, \dots, F_m$. Location in forbidden gaps is not allowed. Denote the centers of X_i and F_j by x_i and b_j respectively; the lengths of X_i and F_j by l_i and p_j respectively; $w_{ij} \geq 0$, $u_{ik} \geq 0$ — the specific costs of connections between X_i and F_j, X_i and X_k. The target is to locate $X_1, \dots, X_n$ on the line outside $F_1, \dots, F_m$ and so that they do not intersect with each other and the total cost of the connections between facilities among themselves and facilities with gaps is minimized. The mathematical model of the problem is [1]:

$$G(x_1, \dots, x_n) = \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij}|x_i - b_j| + \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} u_{ik}|x_i - x_k| \to \min, \quad (1)$$

$$|x_i - b_j| \geq \frac{l_i + p_j}{2}, \quad i = 1, \dots, n, \quad j = 1, \dots, m, \quad (2)$$

$$|x_i - x_k| \geq \frac{l_i + l_k}{2}, \quad i, k = 1, \dots, n, \quad i < k, \quad (3)$$

$$\frac{l_i}{2} \leq x_i \leq LS - \frac{l_i}{2}, \quad i = 1, \dots, n. \quad (4)$$

The problem (1)–(4) is NP-hard and it is reduced to series discrete subproblems [1]. In [2] a definition of local optimum of the problem and variants of lower bounds of goal function of the subproblems are proposed.

In this paper a branch and bounds algorithm for solving of the problem is developed. A computational experiment with use of this algorithm and the approximate algorithm from [1] is made.

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[1]This work was supported by grant 16–01–00740 from the Russian Foundation for Basic Research.

On Solving Academic Load Distribution Problem

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To solve problems arising in the field of education, for example, time-tables scheduling, assigning disciplines to teachers or teachers to schools, determining optimal tests of knowledge checking [2], etc., the models and methods of integer linear programming (ILP) are successfully applied. In particular, the problem of distribution of the academic load, in which the average number of disciplines given to each teacher is minimized, was considered [1]. The proposed mathematical model is a special case of the fixed charge transport problem. In addition, an equivalent combinatorial formulation of this problem was presented, its NP-hardness was proved, and a branch and bound algorithm was proposed. The disadvantage of this formulation is the fact that the load associated with a separate discipline can be allocated to a teacher in an arbitrary volume.

We propose to modify the problem of distribution of teachers academic load as follows. Each academic course consists of indivisible units, that should be given to one teacher. Moreover some other constrains are taken into account, for example, the maximum and the minimum possible amounts of load assigned to a teacher, in accordance with the share of the rate that he holds. Minimization of the maximal number of disciplines assigned to each teacher or the maximization of the total preference of "teacher-discipline" relationship assignments considered as optimization criterion. We show that the problem of finding a feasible solution is NP-hard.

ILP models are constructed, for one of which an L-class enumeration algorithm is proposed. The computational experiment with ILP methods is carried out on random instances and real data problems.

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[1]This research is supported by grant 16-01-00740 from the Russian Foundation for Basic Research.

Analysis and Solving the Problems of Production Groups Formation[1]

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Keywords: operations research, mathematical modeling, discrete optimization, integer programming, optimization on graphs, production groups

The successful activity of a modern enterprise is substantially determined by the effectiveness of staff selection and formation of various kinds of functional groups. Creation of such groups requires consideration of various factors depending on an activity of the groups. For example, when forming production groups, it is necessary to make appointments to posts that ensure the quality and timeliness of work performance, observance of working conditions, accounting for interpersonal and hierarchical relations in the team and other requirements. In the study and solution of such problems, models and methods of discrete optimization are applicable.

In this article, the tasks of forming production groups are considered, taking into account interpersonal, logical and resource constraints [1-2]. For these problems, integer programming models are built, questions of their complexity are studied, and algorithms for their solution are proposed. One of them is based on the branch and bound method, and the search for the optimal solution is reduced to analysis and solution of a sequence of assignment problems. Branching is made based on fulfillment of conditions of consistency of interpersonal and logical restrictions between applicants. The software implementation of the algorithm was done, and a computational experiment was carried out. The purpose of the experiment was to analyze the algorithm and compare it with the CPLEX optimizer. Initial data for test suites was generated randomly. Obtained results showed the potential of the algorithm for further applications.

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[1]Supported by the Russian Foundation for Basic Research, grant 16-01-00740 A.

OPTIMAL CONTROL

An Approach to Solving Control Problems of Heat Processes with Phase Transitions

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The class of problems in which a material under analysis transforms from one phase into another with heat release or absorption is of great theoretical and practical interest. Such problems arise in studies of many phenomena, among which melting and solidification are the most important and widespread. The problems arising in practice do not reduce to the description of processes involving phase transitions, but also include optimal control of these processes. Optimal control of processes involving phase transitions is interpreted as the choice of some process parameters (controls) in such a way that the process is as close as possible to a given scenario; for example, the behavior of the liquid-solid phase boundary or a function of temperature in some domain is closest to a required behavior. An effective approach to solving this type of problems was developed and applied in practice by the authors of this article. The efficiency of the method is explained by the simultaneous use of three basic elements.

First, during the solution of the initial boundary value problem that describes the process of heat transfer, the statement of a boundary value problem in terms of temperature is reformulated in terms of enthalpy. The reason for this is the fact that, as one intersects the phase boundary, the temperature changes smoothly while the enthalpy undergoes a jump change.

The second element of this approach is a special iterative algorithm proposed by the authors for solving nonlinear systems of finite-difference equations obtained as a result of approximating the initial-boundary value problem. The new iterative algorithm is much more efficient than algorithms used earlier: the modified Jacobi method and the modified Gauss-Seidel method.

Optimal control problems for thermal processes with phase transitions are usually solved numerically using gradient methods. To ensure the efficiency of a gradient method, the gradient of the cost function has to be computed to high accuracy. The third element of the proposed approach is connected with the fact that the gradient of the cost function of the optimal control problem is calculated using the Fast Automatic Differentiation technique. This method offers canonical formulas that produce the exact value of the gradient in a discrete optimal control problem.

The above-indicated approach is illustrated based on the example of the solution of the problem of controlling the phase boundary evolution in the substance solidification process in foundry practice. A mathematical model of this process is underlain by a three-dimensional unsteady two-phase initial-boundary value problem of the Stefan type.

A numerical method for time-dependent anisotropic eikonal equations

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In this note we consider the first order scalar PDE

$$H(x,u,\nabla u) = \|A(x,u)\nabla u\| - 1 = 0, \quad x \in \Omega \subset \mathbb{R}^n, \tag{1}$$

with the initial condition

$$u(x_0) = 0, \tag{2}$$

where $(n \times n)$ matrix $A(\cdot,\cdot)$ satisfies the inequality

$$\det A(x,u) > \eta > 0 \quad \forall x \in \Omega, \quad u \geq 0.$$

Though the above equation contains no time variable, it can be viewed as an anisotropic "time-dependent" eikonal equation. Indeed, one may show that any map $u = u(x)$, whose sublevel sets $\{x \mid u(x) \leq t\}$ coincide with the reachable sets $\mathcal{R}(t)$ of the differential inclusion

$$\begin{cases} \dot{x} \in [A(x,t)]^{-1} B, \\ x(0) = x_0, \end{cases} \qquad B = \{v \mid \|v\| \leq 1\},$$

must be a viscosity solution of the problem (1), (2).

The proposed numerical scheme is based on the Fast Marching Method (FMM) [1,2], which proved to be very effective for solving classical eikonal equations. Note that the convergence of the finite difference scheme for the problem (1), (2) requires a monotonicity of the hamiltonian [3]

$$\frac{\partial H}{\partial u} > 0, \quad u > 0.$$

Acknowledgments. The research is partially supported by the Russian Foundation for Basic Research project 17-01-00733 and Comprehensive Program of Basic Research of SB RAS "Integration and Development" project 2017-II.2P/I.1-2.

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On the set of optimal initial conditions in the Dai-Schlesinger's algorithm

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Dai-Schlesinger algorithm often called EM-algorithm in the scientific literature, for calculating the unknown parameters of the Gaussian mixture available sample. To determine the parameters of the mixture are used well-known classical methods, modification of these methods and implement their programs. To calculate the optimal parameter estimates multidimensional blend the mo st effective and commonly used algorithm is the Dai-Schlesinger, based on two methods: maximum likelihood and Picard successive approximations. Use in various fields of science and practice of the Gaussian mixture model with equal covariance matrices and vectors with different mean values due to its resistance to violations of the assumptions of normality and completeness of the system of Gaussian functions in space $L_2(-\infty, \infty)$.

A number of studies with reference to the source noted that the probability P optimal solutions for algorithm Dai-Schlesinger at random the initial conditions dramatically decreases with increasing dimension p of the sample space. However, experimentally is found that the probability P is a decreasing function parameter p, k, ε, (k – the number of components of the mixture, ε – accuracy of calculations), and an increasing function of the parameter ρ_{is}, n (ρ_{is} – Mahalanobis distance between the components of the mixture, $i < s$, $i, s \in \{1, 2, \ldots, k\}$, n – sample size). It should be approved on the basis of analysis of experiments.

When $k \geq 2$ any dimension p of the sample space there exist $\rho_0 = \rho(k, p)$, $n_0 = n(k, p)$, ε_0 ($0 < \varepsilon_0 < 10^{-8}$) that, for all $\rho_{is} \geq \rho_0$, $n \geq n_0$, $0 < \varepsilon < \varepsilon_0$ likely to get the optimal solution $P \geq 0,5$.

In addition, in this study it adjusted the rule for choosing the optimal solution that takes into account the frequency of each solution at a random initial conditions.

Optimal control for discrete-time stochastic systems w.r.t. the probabilistic performance index

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Abstract. The optimal control problem for discrete-time stochastic systems with probabilistic performance index is considered. New results of qualitative research based on the dynamic programming are presented.
Keywords: the discrete-time stochastic systems, the optimal control, the probabilistic performance index, the dynamic programming

The problem in question of this report is stochastic optimal control of discrete-time system w.r.t. the probabilistic performance index. Such models arise in the aerospace, economics and robotics. The existing numerical methods for solving such problems are ineffective because of the known curse of dimension.

The probabilistic performance index is defined as probability that a certain precision functional does not exceed a certain admissible level. Here the precision functional itself characterizes the accuracy of the control system but depends on the trajectory of the stochastic system. One example of such a precision functional is the terminal miss of a guidance system.

In this report we present new results concerning of properties of the Bellman function on the basis of utilisation of the boundedness of the probability.

Using the dynamic programming and the properties of the Bellman function we find two-sided bounds on the Bellman function under general assumptions about the control system, the domain of feasible controls, the precision functional and the random noise distribution. It is proved that under certain conditions the solution of the original control problem coincides with one of the stochastic programming problem of a certain structure.

As an example, the optimal control problem of a portfolio of securities with one risk-free and a given number of risk assets is considered. Using a two-sided estimate of the Bellman function, we prove the asymptotic optimality of the risk strategy.

Acknowledgements. The research was supported by the Russian Science Foundation grant 16-11-00062.

Feedback Minimum Principle for Optimal Control Problems with Terminal Constraints

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Abstract. In a series of previous papers, the author obtained nonlocal necessary optimality conditions for free-rightpoint classical and non-smooth optimal control problems. Feedback Minimum Principle is one of these optimality conditions, which is formulated completely in terms of objects from the Maximum Principle but efficiently strengthen it. In the present talk, the Feedback Minimum Principle is extended to problems with terminal constraints.
Keywords: Optimal control, perturbed Maximum Principle, feedback controls, terminal constraints, modified Lagrangians.

The talk is devoted to generalization of the Feedback Minimum Principle to smooth terminally-constrained problems of optimal control.

The proof of the main result is based on releasing of the constraints by techniques, which provide the property of global convergence, namely, the Lagrange multipliers with quadratic penalty, and parameterization of the cost functional. Both these techniques let us obtain certain versions of the Feedback Minimum Principle in a perturbed (relaxed) form: an optimal process of the addressed problem should be ε-optimal for a sequence of approximation problem of a sufficiently large index.

In the talk, we discuss some aspects of practical implementation of the raised approximate optimality principles, and the related theoretical issues.

[1]The research is partially supported by the Russian Foundation for Basic Research project 17-01-00733 and Comprehensive Program of Basic Research of SB RAS "Integration and Development" project 2017-II.2P/I.1-2.

Relaxational Methods with Feedback Controls for Discrete and Impulsive Optimal Control Problems

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We consider nonlocal optimality conditions based on feedback control variations as well as control improvement methods for discrete and impulsive optimal control problems. The considered problems include measure-driven bilinear control systems with trajectories of bounded variation. Presented optimality conditions strengthen the corresponding Pontryagin Maximum Principle for these control problems. We adapt the approach [2] based on applying functional-descent feedback controls generated by weakly monotone solutions of the corresponding Hamilton-Jacobi inequality under a certain boundary condition. As a connected result, we derive a certain form of duality for the considered problems and propose the dual versions of the necessary optimality conditions (for ordinary and discrete optimal control systems see [2, 3]).

Acknowledgments. The research is partially supported by the Russian Foundation for Basic Research project 17-01-00733 and Comprehensive Program of Basic Research of SB RAS "Integration and Development" project 2017-II.2P/I.1-2.

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On a problem of parametric control of trajectories ensemble

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Keywords: trajectories ensemble, optimal control, reachable set.

The work is concerned to the task of optimal control of trajectories ensemble (or beam) of nonlinear dynamic system. Such problems arise when solving applied control problems of an object with incomplete information [1]. Problem statement for ensembles of trajectories differs from those for a single trajectory, the target functionals depend on the sets. Optimal control for the ensemble is a computationally hard, so we propose to combine numerical methods of reachable set approximation and optimal control methods. Let us consider dynamical system on time interval $t \in T = [t_0, t_1]$

$$x = f(x(t), w(t), v, t) \tag{1}$$

where, $x = (x_1, ..., x_n)$ is phase vector, w is external disturbance limited as $w \in W = [\underline{w}, \overline{w}]$, and v is control parameters vector $v \in [\underline{v}, \overline{v}]$, initial conditions is defined $x(t_0) = x^0$. Suppose $f(x, w, v, t)$ to be continuous differentiable.

In this paper, the ensemble of trajectories will be generated by the disturbance, i.e. the ensemble with the control parameters $\hat{v}$ we call the family of trajectories $x = x(x_0, w(t), \hat{v}, t)$ corresponding to a various realizations of the disturbance w from a given set W. Section of ensemble we call $M_(t, v)$ by the system (1) and control v at time t. At the moment we limits examined problem by only two statement: standard and maximum deviation.

We propose an approach to the solution of the problem based on the approximation of the ensemble section with the use of algorithms developed for reachable set approximation. The most promising methods seem to be represented by stochastic and piecewise linear boundary approximation. The value of terminal functional evaluated on the set of obtained reachable points. Computational experiments showed the principal possibility of solving the problem when using a sufficiently dense coating to a variety of disturbances, and the adequacy of the approximate calculation of the functional in this case.

Acknowledgments. This work is partly supported by the Russian Foundation for Basic Research (project N 15-07-03827).

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Computational technologies for studying set-valued optimization problems

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The problems of unconditional optimization with a set-valued solution, in our opinion, represent an extremal problems class of current importance, but it is not attracting much attention of specialists. This class includes problems that have a set of solutions for which the value of the function coincides with the global extremum. Among the few works on this subject, monograph [1] should be mentioned, there are a classification of such type functions, a number of basic theoretical results, a search algorithm for global minimum and examples of its use. However, the question of finding the set of all solutions at that stage in the development of numerical methods obviously seemed hopeless and is poorly reflected. Close tasks are considered in the theory of incorrectly posed problems, but the main way used to overcome the problem is to introduce new information into the problem. Usually, it is a regularization of functionals, which also does not involve the search for multiple solutions. Methods for interval analysis can also be used to solve the problems of the class of set-valued functions. But, unfortunately, they require analytic expressions of the function, and the dimensions of the problems are very limited. Specialists say that not more than ten variables can be used (see, e.g. [2]), in our opinion, it is critically small.

However, such problems appears in applications quite often. We can mention the underdefined problems of parametric identification, the problem of finding solutions of nonlinear equations systems, the problem of searching for low-potential atomic-molecular clusters, optimal control problems for transferring the system from point to point etc. The report discusses technologies for approximating a set of solutions, based on the use of irregular grids, so-called "cloud approximations". At first stage, stochastic approaches are used to obtain the primary "cloud". Then, using clustering methods, we separate clusters and evaluate it using implemented software. As an application of the developed technologies we present problems of approximating the level set of a function and the problem of finding a global minimum. The results of numerical experiments are presented.

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Recent advances in power systems stability analysis

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Transient stability assessment is one of the most computationally intensive problems in power systems. At the same time, the ability to analyze the stability in real-time is essential for secure operation of Special Protection Systems. The straightforward approach is based on intensive time-domain simulations of the transient dynamics, its application for large-dimensional systems is computationally exhaustive. On the other hand, direct methods allow fast contingency screening via provable stability certificates [1].

In this work we present the extention of stability assessment technique [2] that generalize the well-known equal-area criterion for multidimensional case. Operating with the family of quadratic Lyapunov Functions constructed by linear matrix inequalities [3] we exploit its variety for adaptive construction of certificates for particular contingency. In particular, we restrict nonlinear terms with sector bounds and exploit the theory of Lurie systems with multiple nonlinearities. Following this approach the phase difference between any pair of connected generators can be bounded independently and we may pose tighter bounds for strongly coupled generators and simultaneously relax the bounds for weakly coupled pairs.

The database of certificates can be calculated in a pre-processing step, then it can be used for online stability guarantees. We compare this technique with a more well-known energy method approach and demonstrate how adaptive change of sector bounds reduces conservatism. Several strategies for synthesis of optimal remedial action schemes will be proposed and analyzed.

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On a linear control problem under interference with a payoff depending on the modulus of a linear function and an integral[1]

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We consider a linear control problem under the action of an interference

$$\dot{x} = A(t)x + \phi B(t)\xi + \eta, \quad x(t_0) = x_0; \quad x \in \mathbb{R}^m, \quad t \leq p \ .$$

Here, p is given end time of the control process; t_0 is given initial time; $\phi \in \mathbb{R}$ and $\xi \in M \subset \mathbb{R}^s$ are control; set M is connected, compact and symmetric with respect to the origin in $\mathbb{R}^s$; interference η belongs to connected compact $Q \in \mathbb{R}^m$; $A(t)$ and $B(t)$ are continuous for $t_0 \leq t \leq p$ matrices.

Admissible control are non–negative function $\phi(\cdot) \in L_q[t_0, p]$ and arbitrary function $\xi : [t_0, p] \times \mathbb{R}^m \to M$. Interference realizes as arbitrary function $\eta : [t_0, p] \times \mathbb{R}^m \to Q$. This definition of the control arises in control problems for mechanical systems of variable composition [1]. For example, the law of variation of a reaction mass is defined as a function of time, and the control affects the direction of relative velocity in which the mass is separated.

A quality index of control is value

$$G(|\langle\psi, x(p)\rangle - C|) + \int_{t_0}^{p} \phi^q(r)dr \ . \qquad (1)$$

Here, $\psi \in \mathbb{R}^m$ is given vector; $\langle\cdot,\cdot\rangle$ is scalar product in $\mathbb{R}^m$; C is given value; $G : \mathbb{R}_+ \to \mathbb{R}$ is given function. The aim of control consists in minimization of guaranteed result of the quality index (1).

The control problem is considered within the theory of guaranteed result optimization [2]. With the help of a linear change of variables, the control problem comes down to a homogeneous differential game [3]. An optimal control existence theorem is proved. Necessary and sufficient conditions are found under which an admissible control is optimal.

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Estimation of Frequency Deviations in Power Network with Primary Frequency Control

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Behavior of power system with generators failures under droop control is considered. It is assumed, that there is no congestion management or inter-area constraints and at the pre-failure state frequencies are at the nominal values.

The model of power transmission network [1] is described by a connected oriented graph. It is assumed, that bus voltage magnitudes are constant, line flows are approximated by DC linear power flows, all loads are assumed to be constant loads. Additionally it is assumed that the system was already Kron reduced in order to eliminate load buses. Reduced graph is denoted $G = (V, E)$, where V is the set of n generator buses, E is the set of m lines. Dynamics of the transmission network is specified by the set of differential equations given below

$$M_i\dot{\omega}_i = -d_i\omega_i + \sum_{j:(j,i)\in E} p_{ji} - \sum_{j:(i,j)\in E} p_{ij} + $$

$$- p_i^M + g_i,\ \omega_i(0) = 0,\ i = \overline{1,n}, \tag{1a}$$

$$\dot{p}_{ij} = b_{ij}(\omega_i - \omega_j),\ p_{ij}(0) = 0,\ (i,j) \in E, \tag{1b}$$

$$T^M\dot{p}_i^M = -p_i^M + \alpha_i,\ p_i^M(0) = 0,\ i = \overline{1,n}, \tag{1c}$$

$$T^B\dot{\alpha}_i = -r_i\omega_i - \alpha_i,\ \alpha_i(0) = 0,\ i = \overline{1,n}, \tag{1d}$$

here variables have the following meanings: $\omega_i,\ i = \overline{1,n}$ are deviations of bus frequencies from their nominal values, $p_{ij},\ (i \to j) \in E$ are active power flows, $p_i^M,\ i = \overline{1,n}$ are mechanic power injections at generators, $\alpha_i,\ i = \overline{1,n}$ are positions of valves.

The purpose of the paper is to obtain estimations for convergence rate and maximal frequency deviations. Mathematical description of methodology is given. Preliminary numerical results are presented.

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Minimizing a sensitivity function as boundary-value problem of terminal control

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Abstract. We consider a finite-dimensional optimization problem of minimizing a sensitivity function under constraints as a boundary-value problem in terminal control problem. A saddle-point method for solving problem is proposed. The method's convergence to solution of the problem in all the variables is proved.

Let us consider a problem of minimizing a sensitivity function on set Y of admissible values of y:

$$\varphi(y) = f(x^*) = \mathrm{Min}\{f(x) \mid g(x) \leq y,\ x \in X \subset \mathrm{R}^n\}, \tag{1}$$

$$y^* \in \mathrm{Argmin}\{\varphi(y) \mid y \in Y \subset \mathrm{R}^m_+\}, \tag{2}$$

where $f(x)$ is a scalar convex function; $Y = \{y \geq 0 \mid G(y) \leq d,\ d \in \mathrm{R}^m_+\}$; $g(x)$, $G(y)$ are vector functions with convex components; d is a fixed vector; X, Y are convex closed sets. Solutions to (1) and (2) form convex closed sets X^*, Y^* with $x^* \in X^*$, $y^* \in Y^*$. Problem (1),(2) plays a role of a boundary-value problem in a dynamic problem of terminal control.

To solve (1),(2) we use a saddle-point approach based on reducing the problem to finding a saddle point of Lagrange function $L(x, y^*, p) = f(x) + \langle p, g(x) - y^* \rangle$. Dual extraproximal iterative method has been used to implement this approach:

$$\bar{p}^k = \pi_+(p^k + \alpha(g(x^k) - y^k)), \tag{3}$$

$$y^{k+1} = \pi_Y(y^k + \alpha\bar{p}^k), \tag{4}$$

$$x^{k+1} = \mathrm{argmin}\left\{\frac{1}{2}|x - x^k|^2 + \alpha(f(x) + \langle \bar{p}^k, g(x) - y^k \rangle) \mid x \in X\right\}, \tag{5}$$

$$p^{k+1} = \pi_+(p^k + \alpha(g(x^{k+1}) - y^{k+1})). \tag{6}$$

Theorem 1 (On convergence of method)**.** *If solution to* (1), (2) *exists then sequence* (p^k, x^k, y^k) *of dual extraproximal method* (3)-(6) *with parameter* α *satisfying the condition* $0 < \alpha < \min\{1/(2|g|), 1/2\}$ *converges monotonically in norm to one of solutions of the problem* (p^*, x^*, y^*) *as* $k \to \infty$ *for all* (p^0, x^0, y^0).

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Synthesis of control system in multidimensional nonlinear objects under uncertainty

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The problem of generalizing the method of analytical design of aggregated regulators [1] to the case of control of a nonlinear object having an incomplete description is considered. It is assumed that the following conditions are right: 1) there is a global stability of the target system for the initial model, satisfying technical requirements and target invariant manifold; 2) target manifold can be defined analytically; 3) all solutions of the initial system are bounded; 4) control object is represented by a system of ordinary differential or difference equations. The mathematical statement of the problem has the following form [2]:

$$\dot{x}(t) = f(x;u), \Psi(x(t)) = 0, J = \int_0^\infty ((\Psi(x(t))^2 + \omega^2(\dot{\Psi}(x(t)))^2)\, dt \to \min$$

The vectors $x, f \in R^n, u \in R^m, m \leq n$ from (1) are the vectors of state and of control, respectively; some of the components of the vector $f \in R^n$ are unknown; $\Psi(x)$ is target macro variable; J is criterion of quality control. Basic provisions of the algorithm for constructing control of an object with incomplete description are follows.

1. Control structure in accordance with the classical method of analytical constructing of aggregated regulators is formed [1].
2. The replacement of the unknown description with the upper bounds of the state in the regulator is carried out.
3. The task of achieving the set of desired states of the object with simultaneous compensation of uncertainty in the description is posed.
4. An algorithm [2] based on the non-linear adaptation method is used, which guarantees the output of the control object to a neighborhood of the given manifold and the asymptotically stable retention of the object in this neighborhood.

The theoretical justification of the stability and robustness properties of the proposed control algorithm are given.

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[1]This work was supported by grant 17–08–00920 from the RFBR

On the optimal non-destructive system exploitation problem in certain general setting

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The report is devoted to the use of a renewable resource based on its withdrawal from the certain system, not leading to its destruction (for example, the harvest in ecology). This problem is very relevant at present [1]. Our aim is to present a unified approach to managing such systems.

Let the mapping F is concave on the nonnegative cone $\mathbb{R}^n_+$ of the space $\mathbb{R}^n$. The purpose of optimization is to obtain the maximum admissible total effect

$$\tilde{c} = \max\{\langle c, x\rangle \mid x \in \overline{U}\}, \tag{1}$$

where $c, x \in \mathbb{R}^n_+$, $x = [x_1, x_2, \ldots, x_n]$, $\langle \cdot\,, \cdot \rangle$ — the scalar product, $\overline{U}$ — the closure of the set $U = \{u \in \mathbb{R}^n_+ \mid X_0(u) \neq \varnothing\}$,

$$X_0(u) = \{x_0 \in \mathbb{R}^n_+ \mid \inf_{t=0,1,2\ldots} x^i_t(x_0, u) > 0 \ (\forall\, i = 1, \ldots, n)\},$$

$$x_{t+1}(x_0, u) = \big(F\big(x_t(x_0, u)\big) - u\big)^+, \quad t = 0, 1, 2\ldots \tag{2}$$

Here $x_t(x_0,u) = [x^t_1(x_0,u), x^t_2(x_0,u), \ldots, x^t_n(x_0, u)]$, $x_0(x,u) = x$, $x^+_i = \max\{x_i,0\}$, $x^+ = [x^+_1, x^+_2, \ldots, x^+_n]$.

Denote by $\tilde{u}$ and $\widetilde{U}$ the optimal vector and optimal set of problem (1), respectively; and by N_u and $\hat{N}_u$ the sets of nonzero fixed points and positive fixed points of the mapping $F_u(x) = \big(F\big(x_t(x_0, u)\big) - u\big)^+$, respectively. The behavior of the iterative process (2) is described by the following way:

a) if $N_u = \varnothing$ then $\lim_{t\to+\infty} x_t = 0$ regardless $x_0 \geq y(F)$ where $y(F)$ is the maximum fixed points of the mapping F;

b) if $\hat{N}_u \neq \varnothing$, then $\lim_{t\to+\infty} x_t = y(u)$, where $y(u)$ is the maximum fixed points of the mapping F_u;

c) $X_0(u) \neq \varnothing$ if and only if $\hat{N}_u \neq \varnothing$.

Thus, the equality $\overline{U} = \{u \in \mathbb{R}^n_+ \mid N_u \neq \varnothing\}$ is valid and the problem (1) can be reduced to the following convex program:

$$\max\{\langle c, u\rangle \mid x = F(x) - u,\ x \geq 0,\ u \geq 0\}.$$

Investigated optimal solutions of this problem for some nonlinear generalizations used in mathematical ecology models.

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Weak Invariance for Impulsive Differential Inclusions

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A measure-driven differential inclusion generated by an impulsive control system with trajectories of bounded variation is considered. The solutions of the differential inclusion are upper semicontinuous set-valued functions with selections of bounded variation [2, 3, 4]. The property of weak invariance of closed sets relative to the differential inclusion is investigated. The weak invariance property deals with conditions under which there exists a set-valued solution starting and remaining in a given closed set C. Definitions and characterizations for weak invariance and preinvariance are presented and discussed. These results are related to the proximal theory [1, 5] and have a form of systems of proximal Hamilton-Jacobi inequalities.

Acknowledgments. The research is partially supported by the Russian Foundation for Basic Research project 17-01-00733 and Comprehensive Program of Basic Research of SB RAS "Integration and Development" project 2017-II.2P/I.1-2.

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Optimal Control Problems with Trajectories of Bounded Variation and Hysteresis

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This report deals with an optimal control problem for measure-driven differential equations with rate independent hysteresis. The hysteresis is modelled by scalar rate independent variational inequality with solutions of bounded variation [3, 5]. We prove an existence solution theorem for the system of measure-driven differential equations with hysteresis. The main result is necessary optimality conditions in a form of generalized maximum principle for optimal solutions of the considered control problem. To prove optimality conditions, we regularize impulsive dynamics by adapt an approach based on the discontinuous time reparametrization [2, 4] and some approximation of the variational inequality by an appropriate differential equation [1].

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Estimates of Transients in Discrete Time Linear Systems

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Analysis of transients in dynamical systems is of great importance, e.g., see [1]. However, typically, it is assumed that the initial conditions are zero, and the system is subjected to exogenous input signals. Not much work has been performed for *input-free* systems with *nonzero* initial conditions; e.g. see [2] for a recent publication. We say that a stable system experiences peak if its solution deviates from the unit-norm initial conditions at finite time instants before converging to zero.

The situation with discrete-time systems is different. Though the stability theory for difference equations is well-developed, see [3], to the best of our knowledge, no attention has been paid to the analysis of similar peak effects.

In this paper we consider discrete-time linear systems in the form $x_{k+1} = Ax_k,\ \ x_k \in \mathbb{R}^n$, and discuss several results in this direction. First, for Schur stable systems we provide upper bounds on deviations of trajectories with arbitrary initial conditions of unit Euclidean norm; equivalently, we estimate the peak of $\|A^k\|$. Second, admitting for the presence of a control input, we design a linear feedback that minimizes the peak of the closed-loop system. These two estimates are obtained via use of linear matrix inequalities and solutions of semidefinite programs.

Next, we consider systems with companion-form Schur stable matrices (equivalently, stable scalar difference equations of nth order) with equal real eigenvalues $\lambda_i \equiv \rho$, $\rho \in (0,\ 1)$, and initial conditions $(0, 0, \dots, 1)$ and obtain the exact values of the magnitude of peak and peak instant. We show that, as the order of the system grows and/or the value of ρ increases, the magnitudes of both peak and peak instant grow. The results of numerical experiments are also presented.

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[1]This work was supported by grant 16-11-10015 from the Russian Science Foundation.

Sufficient optimality conditions for extremal controls in optimal control problems

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In given report we consider an optimal control problem for a functional with a convex terminal function. Sufficient optimality conditions are obtained on the basis of nonstandard formulas of the functional increment, which are commonly used to construct numerical methods for successive improvement of admissible controls. The definitions of strongly extremal controls for each formula are introduced. These controls maximize the Pontryagin function for the set of phase or conjugate trajectories. In linear and quadratic problems strongly extremal controls are optimal. In the general case, the optimality property is provided by the concavity of the Pontryagin function with respect to phase variables. Examples of effective realization of obtained conditions in comparison with known results are given.

Impulsive Behavior and Hybrid Properties of Control Mechanical Systems

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Abstract. The talk presents some recent results in the area of modeling and optimal control for a class of specifically mixed-constrained impulsive dynamical systems driven by Borel measures.
Keywords: Hybrid systems, impulsive control, Lagrangian mechanics, trajectory relaxation, approximation, optimization.

We address control dynamical systems described by measure differential equations of the form

$$dx = f(x)dt + G(x)d\mu.$$

Impulsive effects are regarded to be performed by vector-valued Borel measure μ, and states are functions of bounded variation.

The interface between states and the control measures is presented by mixed conditions of a complementarity nature being constraints on one-sided limits of a state before and after its jumps:

$$x(t^-) \in \mathcal{Z}_- \text{ and } x(t) \in \mathcal{Z}_+ \quad |\mu|\text{-almost everywhere},$$

where $|\mu|$ is the total variation of μ.

A practical motivation emerges from Lagrangian mechanics and contact dynamics as a modeling challenge for such effect as impulsive forces of unilateral contact of rigid bodies, or impactively blockable degrees of freedom.

Our goals are: 1) to invent a constructive description of the closure of the trajectory tube for the stated model; 2) design an approximation of the mixed-constrained measure-driven system by ordinary control processes driven by measurable bounded controls, and 3) establish an equivalent transformation of a related optimal control problem to an ordinary variational model.

In conclusion of the talk, we discuss an application of the obtained theoretical results to numerical simulation of hybrid systems.

[1]The work is partially supported by RFBR, grants 16-31-60030, 16-31-00184

On feedback maximum principle for dynamical systems driven by vector-valued measures

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Abstract. The talk presents a variational necessary optimality condition in the form of the feedback maximum principle due to V.A. Dykhta for a class of terminally constrained dynamical systems of a specific structure, originated in impulsive variational problems with control vector measures and states of bounded variation.
Keywords: Optimal control, impulsive control, necessary optimality conditions, feedback maximum principle

Given $T,\ y_T > 0,\ c,\ x_0 \in \mathbb{R}^n$, and globally Lipschitz continuous functions $f:\ \mathbb{R}^n \to \mathbb{R}^n$, $G:\ \mathbb{R}^n \to \mathbb{R}^{n\times m}$, we state the optimal control problem (P):

$$I[\sigma] = \langle c, x(T)\rangle \to \min \ \text{subject to}$$
$$\dot{x} = (1 - \|v\|)f(x) + G(x)v, \quad x(0) = x_0, \tag{1}$$
$$\dot{y} = 1 - \|v\|, \quad y(0) = 0, \quad y(T) = y_T. \tag{2}$$

A triple $\sigma = (x, y, v)$ is said to be a control process. Let $H = H(x, y; \psi, \xi; v)$ denote the Pontryagian (maximized Hamiltonian) of (P), and (ψ, ξ) the dual of (x, y), being the solution to the adjoint system: $\dot{\psi} = -\frac{\partial}{\partial x}H, \quad \psi(T) = c,\ \xi = \text{const} \in \mathbb{R}$. Introduce the parameterized multivalued map $V_\xi = V_\xi(t, x, y, \psi)$ defined as follows: $V_\xi = \{0\}$, if $y \leqslant t - T + y_T$; V_ξ is the unit sphere in $\mathbb{R}^m$, if $y \geqslant y_T$, and $V_\xi = \operatorname{Arg\,min}\{H|\ \|v\| \le 1\}$, otherwise.

Given a reference process $\bar{\sigma}$, let $\bar{\psi}$ be the respective adjoint state. Denote by $\mathcal{V}_\xi$ the set of single-valued selections w of V_ξ restricted to $\bar{\psi}$, and by $\mathcal{X}(w)$ the set of all solutions – in the senses of Carathéodory and Krasovskii-Subbotin – to system (1), (2) closed-looped by feedbacks w.

Theorem. The optimality of $\bar{\sigma}$ for (P) implies that

$$I[\bar{\sigma}] \le \langle c, x(T)\rangle \ \ \forall\, x \in \mathcal{X}(w),\ w \in \mathcal{V}_\xi,\ \xi \in \mathbb{R}.$$

In the talk, we discuss some consequences of the theorem, and its application to numerical implementation of impulsive control problems.

[1]Partially supported by RFBR, grant 16-31-60030
[2]Partially supported by RFBR, grants nos 16-31-00184, 17-01-00733

The Technique of Rational Convexifying for Nonlinear Controlled Dynamical System

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Keywords: optimal control problem, nonlinear controlled dynamical system, global optimization, reachable set.

In this paper, we propose a computing technique for optimization of controlled dynamical system based on the procedure of rational "convexifying" the initial nonconvex optimal control problem. This approach aimed at overcoming the effect of "multiplication" of controls by using of Gamkrelidze's method [1].

The optimal control problems have the property of hidden convexity (A.F. Filipov, V.M. Tikhomirov, A.A. Tolstonogov), which can be used to construct specialized methods for their study. It consists in the possibility of convexifying a velocities set of a controlled dynamical system according to the method proposed by R.V. Gamkrelidze, and the transition to an extended problem with auxiliary controls satisfying given conditions. With the use of this procedure, it is possible to obtain a lower estimate of the global extremum of the objective functional, and in some cases, to convexify the reachable set. In order that any trajectory optimal in the initial problem be optimal in the extended one, it is necessary and sufficient to perform the correctness property by extension: the lower bounds of the functional in the initial and extended problems must coincide.

We developed several ways to reduction the extended problem to the standard form and variants for accounting constraints on auxiliary controls, based on the use of penalty functions, modified Lagrange functions and other approaches. We have identified a class of nonlinear optimal control problems for which, after carrying out the procedure of convexifying, the reachable set becomes convex. The computational experiments carried out to study the properties of the developed technique with test collection [2] made it possible to verify their effective applicability for the study of multiextremal optimal control problems.

Acknowledgments. This research was supported by Russian foundation for basic research, grant No 17-07-00627.

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EQUILIBRIUM AND BILEVEL PROGRAMMING

Welfare impact of bilateral tariffs under monopolistic competition

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Some bilateral tariffs remain even under WTO. They induce a question for theorists: Can any mutual tariff be welfare enhancing? A positive–protectionist–answer is known under some oligopolistic or dynamic hypotheses, see Brander & Spencer (1984). However, the question remains insufficiently studied in “new” trade theory, based on monopolistic competition. Unlike oligopoly, here strategic behavior of firms cannot be a reason for protectionist thinking. Still, some welfare gains from tariffs look generally possible, because under variable elasticity of substitution the equilibrium number of firms can be socially excessive or insufficient. So, mutual tariffs can be thought off as a remedy for such “distortion of variety.” However, tariffs generate a “structural distortion:” asymmetry between consumption of domestic and imported goods. Therefore, overall welfare impact of tariffs is non-obvious. This paper supplements related theory under Krugman’s trade setting. The simple setting focuses on Krugmanian market forces, unlike selection or inter-sectoral effects. We find analytically that any bilateral tariff makes the equilibrium consumption of each domestic variety growing, import decreasing. Simultaneously, total output of a firm decreases, because a monetary transfer from tariffs stimulates the domestic consumption insufficiently to compensate decreasing import. Surprisingly, variety (mass of firms), increases in tariff, because variety must be inversely related to each firm’s output under free entry. In the (realistic) case of decreasingly elastic utility, we prove that for additive preferences any small tariff reduces social welfare in both countries because both kinds of distortion are summed up, but any subsidies for imports or export tariffs bring a symmetrical outcome. I.e., social welfare increases with small subsidies. To roughly quantify the losses and gains, we follow the methodology of Melitz & Redding (2015), and consider compensating variation. Using calibrated trade elasticity from the CES-literature, we apply, however, the two-parametric AHARA utility function (flexible enough for our goals).

Interaction of Consumers and Power Supply Company for Demand-side Management

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The research proposes a methodology for the formation of tariffs in the retail electricity market. The methodology takes into account several factors that affect the demand-side management, namely: the presence of various types of consumers; the possibility of selecting a tariff type from several tariffs proposed by power supply company; the need to stimulate consumers to optimize their load during a day, in particular, to smooth peak load by shifting it to off-peak time; and the elimination of shortage risks. In our research the shortage is considered as a deviation of the actual consumption from the planned load, and, hence the emergence of the need to purchase the deficient electricity at higher prices. Interaction of consumers and power supply company is presented as a non-cooperative game [1].

A distinguishing feature of our research is the differential approach to stimulation of different consumers to optimize their load during a day. The problem of active consumer is solved by dividing all consumers into fully rational (their actions may be described by a financial result only) and boundedly rational (less motivated to save, whose strategy of behavior includes the notion "convenience"). Consideration of boundedly rational consumers is an important step in the research of such kind. We describe the behavior of these participants in the interaction through the utility function of a special form [2]. In our research the problem is maximization of the power supply company's profit with constraints which describe the utility maximization for several types of consumers in a simplified form. The existence and uniqueness of separating equilibrium for the two types of consumers with different utility functions are proved.

Acknowledgments. This work was supported in part by the grant 16-06-00071 from RFBR.

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Upper Bound for a Problem of Competitive Facility Location and Capacity Picking in a Case of Multiple Demand Scenarios[1]

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The paper considers a bilevel mathematical model where two competing parties sequentially open their facilities with the aim to capture customers and maximize profit. One of the parties, called a Leader, opens its facilities first, and after it, knowing Leader's decision, another party, called a Follower, opens its facilities at the second turn. Customer capture depends on customer's preferences which are given by linear order on the set of potential locations. The party captured the customer can assign only those facilities to serve him or her which are more preferable for the customer than any competitor's facility.

In the model under consideration, multiple demand scenarios are possible, but only one of them is to be realized. That scenario is revealed after Leader's turn and before the Follower's one. Thus, Follower makes its decision knowing both Leader's decision and the set of customers with all their attributes. We assume that operating cost of the facility is a sum of its fixed cost and a term which is proportional to the facility's capacity. Leader's goal in the competition is to determine the set of open facilities and their capacities so that that its profit is maximized in a worst scenario while Follower acts rationally by maximizing own profit in each scenario as well.

The present paper presents further progress of the method of estimating problems construction what enables us to calculate upper bounds for competitive location problems and develop methods to solve them optimally. The method consists in formulating additional constraints of Leader's problem which improve high-point problem estimations. The technique was introduced in [1, 2] and extended by adding new constraints improving the quality of the upper bounds obtained.

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[1] The research is supported by Russian Foundation for Basic Research (project 15-01-01446).

Monopolistic Competition Model with Different Consumer Utility Levels

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We consider a monopolistic competition model with endogenous choice of technology in the closed economy case. The aim is to make comparative statistics of equilibrium and social optimal solutions with respect to «consumer utility level» parameter β which influences on utility.

Key findings: when parameter β changes,

(1) the behavior of the equilibrium (social optimal) individual investments in R&D, individual consumption, and mass of firms depend on the behavior of the demand (respectively, utility) elasticity;

(2) the behavior of the equilibrium (social optimal) total investments in R&D depends on the behavior of the elasticities of both demand (respectively, utility) and marginal costs.

The paper concerns with [1]. Also we discuss the generalization the results to another monopolistic competition models: retailing [2], market distortion [3], international trade [2], and to the marketing models: optimization of communication expenditure [5] and the effectiveness of advertising [6].

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Harmful Pro-Competitive Effects of Trade under Classical Monopolistic Competition

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Gains from trade is an evergreen topic. In New Trade, it again attracted a discussion after [1] puzzled the theorists with surprisingly low estimated gains. Moreover, even welfare loss may occur in free trade in comparison with autarky [2]. By contrast, this paper discovers harmful trade under additive VES utilities enabling autarky - but only under very high trade costs, near autarky: the first step from autarky is harmful. The setting is close to classical Krugman's trade model, it includes one diversified sector, no outside good, and unspecified additive utilities. Homogeneous firms use one production factor (labor) with same fixed and marginal cost, consumers are identical. Labor and trade are balanced. Near free trade, welfare locally increases in each country with liberalization (under realistic, decreasingly elastic utilities). More subtle and unexpected is the effect at the beginning of liberalization, near autarky. From the policy viewpoint, our "harmful trade" seems to favor protectionism, but in fact, it only suggests not to liberalize trade gradually, to jump over the initial losses towards sufficiently massive trade. See [3] for details.

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Game Theory Approach to Malfatti's Problem

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Keywords: alfatti's problem, Nikaido-Isoda function, nonconvex optimization.

We consider Malfatti's problem from a view point of Game theory. The problem reduces to a generalized Nash equilibrium problems with shared constraints. For solving the problem numerically, we use Nikaido-Isoda function and penalized techniques for nonconvex constraints.

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Public-private partnership model with tax benefits

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In [1, 2], a new approach was proposed for design the program of development of the mineral resource base by using the mechanism of public-private partnership. The private investor was going to implement some investment projects that require preliminary infrastructure work and take into consideration environmental losses (pollution of rivers, lakes, etc.). The state took on infrastructure and part of ecological problems. The mechanism of public-private partnership is based on the "leader-follower" Stackelberg game and bilevel Boolean programming model, where the leader is the state.

In this paper we consider the possibility of tax benefits for any investment project by the state. We show results that characterize the computational and the approximational complexity of the problem. We design a hybrid algorithm based on local search and CPLEX software. Numerical experiments are conducted on special polygon of test instances based on the mineral resources of the Transbaikal territory.

Acknowledgments. This work is supported by the Russian Humanitarian Foundation (project 16-02-00049).

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Minimization of borrower's payments in mortgage lending

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In [1], the problem of minimizing the borrower's risks in mortgage lending was considered. The model described gives good practical guidelines for reserving funds. In this paper, we describe a model in which, in the case of a shortage of funds for the current payment, a default is not declared, as in the model from cite MS, and the borrower has additional costs. Such costs can be accounted for using short-term ancillary loans. Then there is no risk of default on the main debt, but additional loans can significantly increase the total payments that need to be minimized.

The borrower wants to buy an apartment cost S_0 and he has accumulation of M_0 rubles. Out of this money, a reserve of Z_0 is formed. The difference between M_0 and Z_0 is paid as an initial contribution, and $D = S - M + Z$ is a credit for an annuity scheme for T years at a rate r. The borrower's income at the moment of time t is a random variable $xi(t)$ with a known distribution.

In the presence of free cash, the borrower may early repay part of the loan. If current receipts are not enough to pay off the next payment, the borrower takes an additional loan for a single period of time at a rate of $r_B > r$. Within the framework of this model it is we need to determine the total credit period T and the amount of reserved funds Z_t, at which the expectation of total payments on loans will be minimal.

To solve this problem, an algorithm based on the dynamic programming scheme has been developed. A program was written and an experiment was conducted, including real data. Dependencies of payments on parameters T and Z_0 are constructed, the theoretical justification of the obtained results is carried out. Calculations allow us to identify the critical values of the parameters T and Z_0, at which the total payments are significantly increased.

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Maximizing the profit of a logistics company with limited capital

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The trading company buys goods in bulk and sells them in the retail market. For each product $i = 1, 2, \ldots, N$ the following parameters are known: λ_i – intensity of sale; $\alpha_i + \beta_i v_i$ – cost ordering and delivery of a batch of v_i; c_i – the unit cost of the sale; c_i^{xp} – the cost of storing a unit of goods per unit time. The optimal period T_i^* and the corresponding import volume $v_i^* = T_i^* \lambda_i$ is determined from the condition for maximizing the specific net reduced the profit $U_i(T_i) = \frac{1}{T_i}(\int\limits_0^{T_i} \frac{c_i \lambda_i}{(1+r_0)^t} dt - \alpha_i - \beta_i T_i \lambda_i - \int\limits_0^{T_i} \frac{(T_i \lambda_i - t\lambda_i) c_i^{xp}}{(1+r_0)^t} dt)$, where r_0 – the norm of alternative risk-free allocation of liquidity of capital [1].

Currently, in most firms that sell a wide range of goods, orders are made automatically without the participation of employees of the company. At the next order the program uses current values of parameters of intensity, cost and so on. In practice, there are situations when the program calculated by the program robot is not provided with finances. This is possible during the payment of taxes, the diversion of some amounts to external or domestic investment. Due to the lack of working capital, the application has to be reduced. With several hundred or even thousands of positions in the nomenclature, this can not be done manually. The model is constructed and algorithms are developed to automate the process of minimizing losses while reducing the application, taking into account the possibility of using short-term loans.

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Exact method for the competitive facility location problem with quantile criterion[1]

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The location model in the scope of the present paper was firstly introduced in [2] as a deterministic reformulation of a stochastic competitive location problem and can be thought as a multi-scenario generalization of the problem from [1]. It considers two competing parties opening their facilities in a finite discrete space with the goal to maximize their profits. Emulating the framework of Stackelberg game, the decision-making process is organized as follows. One of the parties, called Leader, opens its facilities first. The set of customers is unknown for Leader at the moment of making a decision. Instead of this, Leader is provided with a finite set of possible scenarios. Each scenario has a probability of realization and fully characterizes the set of customers.

After the Leader's turn, one of the possible scenarios is realized and the set of customers becomes specified. This information is available for the second party, Follower, who opens its facilities with the goal to maximize profit. Leader's goal in this situation is to make a profit that can be guaranteed with given probability or *reliability level* as big as possible.

In [3] we formulate an estimating problem in a form of MIP providing an upper bound for Leader's objective function. Two reformulations of the estimating problem are suggested as well. In the present work we develop branch-and-bound algorithm utilizing the aforementioned upper bounds and perform numerical experiments with artificial data to investigate their efficiencies.

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[1] This research is supported by the Russian Science Foundation (grant 15-11-10009).

Equilibrium on a heat market under network constraints

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The mathematical model of the electricity market and heat is defined as follows. As the basis the article [1] was used with the addition of the network structure of the market. Consider power system with I nodes $i = 1, \dots, I$ in which there are participants of market: various types of plants producing electric power P and plants producing heat Q, as well as the agents consuming electric power x and heat y (for details [1]).Each consumer has an utility function F_i^{CE} for agents consuming electricity and F_i^{CH} for heat.

$$F_i^{CE} = c_i^e x_i - r_i^e x_i^2; F_i^{CH} = c_i^h y_i - r_i^h y_i^2, \tag{1}$$

Consider the objective function which is a public welfare of this power system, so the consumers maximize their utility functions and the producers minimize their costs. Therefore we are maximizing the welfare:

$$F = \sum_{i=1}^{n} F_i^{CE} + \sum_{i=1}^{n} F_i^{CH} - F^{COST} \to max, \tag{2}$$

where:

$$F^{COST} = \sum_{i=1}^{n} a_i^{tec}(qP_i^{tec} + Q_i^{tec}) + \sum_{i=1}^{n} a_i^{kes} P_i^{kes} + \sum_{i=1}^{n} a_i^{kot} Q_i^{kot}, \tag{3}$$

We need to write the equations of network balance:

$$x_i = \sum_{i=1}^{n} P_i + \sum_{j=1}^{n} d_{ij}^{Э}(1 - \delta_{ij})v_{ji} - \sum_{j=1}^{n} d_{ij}^{Э} v_{ij}, \tag{4}$$

$$y_i = \sum_{i=1}^{n} Q_i + \sum_{j=1}^{n} d_{ij}^{T}(1 - \delta_{ij})w_{ji} - \sum_{j=1}^{n} d_{ij}^{T} w_{ij}, \tag{5}$$

$$\sum_{i=1}^{n} P_i^{tec} = b \sum_{i=1}^{n} Q_i^{tec}, \tag{6}$$

$$w_{ij} \le wMax_{ij}; v_{ij} \le vMax_{ij}. \tag{7}$$

where:
d_{ij} is a matrix showing the network structure;
v_{ij} is a flow of electric power between nodes ;w_{ij} is a flow of heat power;
δ_{ij} is s the percentage of losses ;
The results of numerical experiments are presented.

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Mathematical model of competitive equilibria on the electricity market of Mongolia

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As known, the modeling of economical conflicts via Game Theory is a natural tool for this purpose [1]. For example, a bimatrix game can be applied for analysis of the duopolies [1, 2, 5]. A greater number of competitors in the oligopolistic market requires the use of at least polymatrix games tool [1, 3].

A problem of competition between three countries (Mongolia, Russia, China) on the electricity market of Mongolia is investigated. Modeling of the conflict is carried out using the apparatus of three person polymatrix games (hexamatrix games). To find a Nash equilibrium in the constructed game we use an approach based on its reduction to a non-convex optimization problem with bilinear structure in the objective function [3]. To solve the latter problem we apply Global Search Theory due to A.S. Strekalovsky [4]. According to the theory, local and global search algorithms for formulated game are developed. Local search method is based on the idea of sequential solving of auxiliary linear programming problems followed from the formulation of the problem. Global search based on a specific Global Search Strategy in the d.c. maximization problems as the objective function of the reduced optimization problem can be represented as a difference of two convex functions [3]. The results of a computational simulation is presented and analized.

Acknowledgments. This work of first two authors is carried out under financial support of Russian Science Foundation (project no. 15-11-20015).

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Quality and Entry Deterrence on Two-sided Market Platform

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In this paper, we aim to contribute to the effort of proposing the model to analyse platform quality competing strategy. We consider the models of two scenarios of two-sided market platforms: one is the monopolistic two-sided market platform, and the other is the incumbent market platform facing a potential entry competitor. There are three strategies (blockade, deter or accommodate the entrant) for the incumbent platform on entry deterrence. We found the quality level threshold (minimum/maximum quality level that would deter/accommodate the entrant) for the incumbent to make a choice on competitive strategies. A two-sided market is two sets of participants interact through a platform and the decisions of each set of participants affects the outcomes of the other set of participants. The two sets of participants do not link themselves directly, instead, the market platform acts as an intermediary of the networks that enables the two sets of participants to transact and interact with each other. The features of the two-sided market do not like traditional markets. The platform provides services on both sides, while an increase in the number of participants will increase the value of the other side of the participants. For the network externality effect, demand for a product or service will create more demand, such as more products on a platform induces more demand for the platform. Competition on paltform are distingiushed as "inside competition" and "outside competition". Inside competition happens within the same platform. For instance, the different sellers compete for a consumer on shopping site. Outside competition happens when the platforms compete to get the two sides of user groups to use their platform, this kind of competition take place between the platforms themselves. A two-sided market can be found in many intermediaries, such as e-shopping site composed of sellers and buyers and the sharing economy platforms for housing (hosts and renters) and transportation (drivers and passengers). In this paper, from the two-sided market platform aspect, we aim to contribute to the effort of proposing the model to analyse platform quality competing strategy.

Three-level models of competitive pricing

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In this paper we consider the three-level model of competitive pricing described in [1]. The model is formulated as a Stackelberg leader–follower–clients game, where two companies (the leader and the follower) assign prices in own facilities successively to service the clients. Each facility produces a homogeneous product. Each client has a budget and a single demand. He selects the facility with minimal total payment (price and transportation cost) and purchase the product if his payment does not exceed his budget.

We assume two spatial pricing strategies: uniform pricing and discriminatory pricing. Under uniform pricing each facility charges identical price. In contrast, under discriminatory pricing each client may be charged a different price. We present exact polynomial-time algorithms to solve the problem with the following pricing cases:
1) the leader and the follower use uniform or discriminatory pricing strategy simultaneously;
2) the leader applies uniform pricing, in contrast, the follower applies discriminatory pricing.
In addition, we discuss some open problems.

Acknowledgments. This work is supported by the Russian Science Foundation (project 17-11-01021).

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Cooperation in Dynamic Multicriteria Games

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Abstract. We consider a dynamic, discrete-time, game model where the players use a common resource and have different criteria to optimize. New approaches to construct noncooperative and cooperative equlibria in dynamic multicriteria games are constructed.
Keywords: dynamic multicriteria games, Nash bargaining solution.

Consider a bicriteria dynamic game with two participants in discrete time. The players exploit a common resource and both wish to optimize two different criteria. The state dynamics is in the form

$$x_{t+1} = f(x_t, u_{1t}, u_{2t}) , \;\; x_0 = x , \tag{1}$$

where $x_t \geq 0$ is the resource size at time $t \geq 0$ and $u_{it} \in U_i$ denotes the strategy of player i at time $t \geq 0$, $i = 1, 2$.
The payoff functions of the players over the infinite time horizon are defined by

$$J_1 = \begin{pmatrix} J_1^1 = \sum\limits_{t=0}^{\infty} \delta^t g_1^1(u_{1t}, u_{2t}) \\ J_1^2 = \sum\limits_{t=0}^{\infty} \delta^t g_1^2(u_{1t}, u_{2t}) \end{pmatrix} , \; J_2 = \begin{pmatrix} J_2^1 = \sum\limits_{t=0}^{\infty} \delta^t g_2^1(u_{1t}, u_{2t}) \\ J_2^2 = \sum\limits_{t=0}^{\infty} \delta^t g_2^2(u_{1t}, u_{2t}) \end{pmatrix} , \tag{2}$$

where $g_i^j(u_{1t}, u_{2t}) \geq 0$ gives the instantaneous utility, $i, j = 1, 2$, and $\delta \in (0, 1)$ denotes a common discount factor.

First, we construct a multicriteria Nash equilibrium using the approach presented in [2]. Then, we find a multicriteria coopearative equilibrium as a solution of a Nash bargaining scheme with the multicriteria Nash equilibrium playing the role of status quo points.

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[1] This work was supported by the Russian Foundation for Basic Research, projects no. 16-01-00183_a and 16-41-100062 p_a.

On a Traffic Assignment Problem with Elastic Balanced Demand

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We study a traffic assignment problem with elastic balanced demand. This problem appeared from the idea to design a cloud service for interactive traffic modeling in a developing urban infrastructure. We formulate this problem as a variation inequality with a non-potential mapping and a feasible set which is determined as balance constraints for the total number of trips originating and terminating in different zones. It is shown that the solutions of the studied problem are Wardrop equilibrium flows in terms of well-defined generalized route travel costs. We establish the uniqueness of the generalized equilibrium travel cost as well as the invariance of the set of shortest routes on the solution set of the studied problem. We propose the economical interpretation of dual multipliers for the balance constraints from the urban infrastructure extension point of view. We suggest an approach to the solution of the problem and conduct computational experiments on a real transportation network.

Acknowledgments. This research was partially supported by RFBR grant 16-07-00340.

Core allocations in a mixed economy of Arrow-Debreu type

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This paper investigates coalitional stability of the equilibrium allocations in a mixed economy of Arrow-Debreu type. A notion of fuzzy domination in a mixed environment will be given, and coincidence of the fuzzy core and equilibrium allocations will be shown to hold under rather ordinary conditions.

An important feature of the mixed economic system under consideration is that two different regulation mechanisms function jointly: central planning and flexible market prices. Thus, this model is characterized by the presence of dual markets. In the first market, prices are stable and the allocation of commodities is determined by rationing schemes and government orders. In the second market, prices are flexible and are formed by the standard mechanism of equating demand and supply. We assume that the excess of any commodity purchased in the first market may be resold by any economic agent at flexible market prices.

To introduce an appropriate notions of ordinary and fuzzy domination in a mixed economic system we have to take into account both the above mentioned presence of fixed prices for rationed commodities and the multiplicity of types of coalition stability of equilibrium allocations, which correspond to different types of flexible prices in the second market. A universal way to overcome the difficulties indicated is based on the use of a piecewise linear approximation of nonlinear income functions and leads to the formation of several types of cores which characterize all possible variants of coalition stability. The result is that the testing of the famous Edgeworth's core equivalence conjecture reduces to analyzing the asymptotic behavior of each core individually. Besides this general setting, we pay strong attention to special classes of mixed economic systems that admit a global linearization of the budget constraints. This fact allows us to limit our considerations to a unique type of coalition stability for all equilibrium allocations. To provide the possibility of linearization "in the large", we introduce rather natural assumption of willingness to buy at the second market. This assumption seems to be quite important in some applications of the mixed economy theory. And, together with some technical requirements, this assumption guarantees the coincidence of fuzzy core and equilibrium allocations of the mixed economy under consideration.

Acknowledgments. The research is supported in part by the Russian Fund for Basic Research (grant 16-06-00101).

Non-stationary model of public-private partnership

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An important part of problems of natural and resource complex of Russia is devoted to development of the mechanism of public-private partnership (PPP). As a rule, the investor cannot start an investment project due to a lack of the necessary infrastructure. On the other hand, the state officials are unwilling to invest in infrastructure without guarantee that it is used efficiently. Practical examples of the solution of this problem are not really successful in the Russian conditions. Thus, we are interesting in the following question: what economic and mathematical tools should be used for designing efficient PPP models in the Russian context?

To describe adequately the interaction hierarchy between the state and the private investors in the natural resources sector, we use the concept of the Stackelberg equilibrium. The relevant economic and mathematical tools are based on the Stackelberg games and the mixed integer bi-level programming models. This approach ensures long-term efficiency for the state as well as for the private investors.

Recently, a stationary PPP models were considered assuming that the start of each project is included in the input data. In this paper, we study so-called non-stationary model. A general technological description of production, infrastructure and ecological projects are input information of the bi-level model. The start of each project is a variable, and the ranges of possible mutual temporary lags of projects of various types are given. As a result, we can find a subset of the most profitable projects of all types and their schedule for the state and investors. We design stochastic local search algorithm for this bi-level model and use CPLEX software for solving the investor's sub-problem at each step of the algorithm. Computational experiments are conducted on special polygon of test instances based on the mineral resources of the Trans-Baikal territory. Our experimental results illustrate that the approach can be used as a basic tool for long range strategic planning of the mechanism of public-private partnership.

Acknowledgments. This work is supported by the Russian Humanitarian Foundation (project 16-02-00049).

APPLICATIONS OF OPTIMIZATION METHODS IN ENERGY PROBLEMS

A bilevel model to improve residential electricity tariffs

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In this work we apply bilevel programming to model pricing incentives in order to optimise residential electricity consumption. In bilevel problems there are two agents, called a leader and a follower, interacting at two levels of a hierarchical structure. In our case the leader is an electricity provider producing energy to satisfy the demand of the residential consumers (the follower). The current hourly electricity consumption is unbalanced resulting in the peaks and, therefore, in involving the costly energy generation technologies. To this end, the leader has to regulate the hourly consumption by pricing strategies. It would like to force the consumers to react on a new tariff resulting in the rational usage of energy.

The leader problem is to define the new tariff maximizing the profit of the company which is the difference between the consumers electricity payments and the energy production costs. The consumers problem is to choose between the existing and the new tariffs to satisfy their demands with minimal payments and inconveniences caused by changing the time periods of electricity usage.

To solve the proposed model we apply a solution approach developed for the linear mathematical programs [1, 2]. It is based on a mixed integer single level reformulation applying duality theory and complementary slackness conditions. We demonstrate the sensitivity of the model to parameter changes on small-size instances, and we test the behavior of the model on the realistic data.

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Optimization Problem for Age Structure of Power Plants Basic Types

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In this study, the modeling is based on the integral model for the development of the electric power system [1, 2]. The model accounts for division of the generating equipment into certain age groups. Each of these groups has various technical and economic functioning parameters which describe the capacity aging processes. In the vector model [3] generating capacities are divided into three types: thermal power plants, nuclear power plants and hydro power plants respectively. The model includes the balance integral Volterra equation of the first kind with variable upper and lower limits and functional equations, describing the structure of the electricity consumption of different types of power plants. These functional equations closed the system of integral-functional equations. Also, the model includes restrictions-inequalities for the annual total growth of installed capacity.

We consider a search problem for the optimal generating equipment lifetime for a given demand for electricity and minimum total costs of commissioning and operation of capacities. Numerical solution of the optimization problem for various economic options is given. All calculations have been made as applied to the Unified Energy System of Russia.

Acknowledgments. The work is supported by Russian Foundation for Basic Research (project 15-01-01425).

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A Matheuristic for a Unit Commitment Problem[1]

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We consider a modification of unit commitment problem presented in [2]. It aims to minimize the total operational cost of the electric power system by determining the set of generating units in use and the amount of energy production of the chosen generating units in each period of a planning horizon. The schedule must meet a number of specific constraints such as technical parameters of each individual generator, regulatory requirements of the power system etc.

The unit commitment problem is of great practical importance and has received a significant attention in the literature. Most of the solution approaches for this problem rely on dynamic programming techniques, Lagrangian relaxations, ordering heuristics, and metaheuristics. The methods based on the mathematical programming become more and more capable as the MIP optimization software and computers rapidly progress. A development of that kind of methods is perspective even for the large-scale instances [1].

In the practice of Russian Unified Power System, several hundreds of generating units are to be scheduled over the planning horizon of 72 one-hour periods. We propose a heuristic method based on the mathematical programming techniques (a matheuristic) to deal with instances of that dimensionality and carry out numerical experiments with data featuring the real-life structure to evaluate its performance.

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[1] This research is supported by the Russian Science Foundation (grant 15-11-10009).

Simultaneous estimation of effective production funds and production functions: on the pattern of productions of energy, gas, and water

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Effective production funds(EPFs) of a region, country, and sectors of economy are a part of balance (inventory)funds participating in creation of goods and services. In the production functions (PFs) representing complex production objects, the effective funds, but not balance ones, have to be used. A method for the construction of standard PFs, which have EPFs as one of factors, was suggested in [2], for the case when production statistics contains investment data instead of or together with capital ones. The estimation of PF's parameters uses the capital dynamic equation defined by investments as well as a depreciation rate and a lag of investment capitalization. The latter values and the initial value of EPFs should be estimated simultaneously with a PF's parameters.

The problem of simultaneous estimation of the PF's and the dynamics parameters is an ill-conditioned one. It should be regularized with appropriate usage of additional expert information and non-trivial optimization technique. In [2] the calculation problems were overcome by the subsequent complication of classes of the PFs under estimation starting from the simplest Cobb and Douglass one. Also in the paper a special variant of the continuation method [3] was suggested which can overcome complexity of nonlinear minimization.

The work [1] develops the approach of [2] in the next respects. A coefficient of the realizability of investments is introduced in the dynamics equation. This coefficient represents a ratio of really used capital investments which is less of one because of corruption. As an additional means for overcoming of computational complexities, the transform to the index form of PFs was used. The latters are the production functions of the same specification which variables are the ratios of the current values of the original variables with respect to their initial values.

In this paper a new regularization condition on the initial and final values of the EPFs is being introduced. Results of realization of the proposedestimation model and techniques to real data for the sector of productions of energy, gas, and water of Russian economy will be presented.

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Semidefinite relaxations for the optimal power flow: robust or fragile?

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The optimal power flow problem (OPF) aims at constructing optimal operating regime of a power system that minimizes generation cost or network losses subject to engineering and physical network constraints. The OPF problem is a part of System Operator's daily routine, however most of the industrial solvers use the linearized (so-called DC) formulation. Original quadratic formulation (AC OPF) is more precise, but harder to solve. Among different optimization approaches we investigate semidefinite relaxations [1], [2] that demonstrated their efficiency for the number of industrial benchmarks. However, for transmission networks there is no theoretical validation of the conditions that guarantee the exactness of relaxation. Motivated by the simple illustrative example [3], we investigate the situations when the relaxed formulation is unable to retrieve feasible solution.

We demonstrate that slight modifications to the objective function and/or constraints can lead to the failure of the relaxation. Particularly, two modified problems were formulated: in the first problem constraints on reactive power were omitted, and in the second one, several buses of the network were allowed to consume unlimited amount of power, but not to generate. For simplicity, we assumed the first bus to be the only generator in the grid and minimized its active power injection under constraints on demand on power and voltages at each bus. These two problems were tested on IEEE14 network. In both cases the rank constraint was violated that marks the failure of semidefinite relaxation.

For the moment semidefinite relaxations for OPF are on the way to get embedded in the industrial software. Numerous papers report zero gap between initial and relaxed problems. Our research provides a step towards justification the optimality of the relaxed solution as well as marks the warnings that should be taken into account.

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Balancing Residential Energy Consumption and End-User Comfort

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With the currently growing trends in residential electricity consumption, maintaining the desired reliability of the energy system is of utmost importance. The stress on the grid can be diminished by inducing lower energy use during the peak-load hours In practice, it can be done by means of controllers that limit energy consumption of individual consumers. However, an ill-conceived disconnection of residential loads can have dramatic implications on end-user comfort. Impossibility to operate domestic devices and the need to alter consumption patterns might significantly deteriorate consumers' satisfaction with an energy provider. Thus, in case of on-site electrical water heating loads, cut-off of electric energy during the periods of hot water usage might result in the increased user thermal discomfort and the following complaints to the utility company. To avoid these situations, there is a need for solutions that allow to balance energy consumption and consumers' comfort.

In this paper we demonstrate how such balancing can be achieved on the level of a single domestic device. We consider a scenario wherein the energy consumption of an electrical tank water heater (WH) is scheduled along a daily timescale to minimize its impact on the energy system while respecting the user comfort. To formulate this problem, we use an LPI-optimization approach and a discrete time model with a daily time horizon. The first objective function f_1 represents a possible user thermal discomfort that (s)he can experience due to electricity cut and that has to be minimized. Our thermal comfort model incorporates parameters such as the water temperature, duration of water usage and personal tolerance to cold water temperature. The second objective f_2 is a daily energy consumption of the WH to be minimized subject to a user-acceptable thermal discomfort (f_1) and some WH's engineering constraints. Given a daily hot water usage profile (e.g., from a forecast), we solve the problem for different comfort levels, which yields Pareto front where each individual solution corresponds to a certain daily plan of heat injections into the tank (energy consumption) and the associated user thermal comfort.

Simulation results for various water usage profiles reveal possible trade-offs between the electric energy reduction and the consumers' thermal comfort. The proposed approach lets utility companies exercise reliability measures more cautiously with awareness of consumer comfort. While consumers can benefit from energy (money [1]) savings and can get an insight about their electricity consumption habits.

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Cyber Security-Oriented EPS State Estimation

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The electric power systems (EPS) in the most advanced countries are developing towards the creation and large-scale adoption of smart grid which got the name of intelligent energy system in Russia. An attribute of the smart grid is cyber-physical intrusion tolerance of the network. Developing the conceptual smart grid models and projects, researchers nowadays, pay great attention to the issue of cyber security. In this connection it is necessary to note the elevated vulnerability of EPS information-communication infrastructure. Thus, it becomes essential to upgrade the existing mathematical tools and develop new ones to furnish the data of required quality to the tasks of EPS control and monitoring under internal and external impacts.

We consider the state estimation tool as a link between physical and information - communication infrastructures of EPS. It acts as a barrier to the corruption of data on current operating conditions of the electric power system in the control problem, including the data corruption caused by cyber attacks on data collection and processing systems of the EPS.

State estimation is a mathematical data processing method which is widely used for calculation of power system state variables on the basis of measurements. The most vulnerable facilities in terms of cyber attack consequences for state estimation are the information-communication control subsystems (SCADA and WAMS). Since the input data for the state estimation are represented by the SCADA measurements and PMU data. Due to cyber attacks on the SCADA and WAMS, measurement data coming to the state estimation problem are distorted. If no special measures are taken to identify these distortions and suppress their impact on the state estimation results, serious errors can appear in decisions made by dispatchers using the state estimation results. Therefore, to obtain quality state estimation results, the used measurements should be tested for the presence of bad data.

Researchers from Energy Systems Institute SB RAS have developed the method of test equations to detect bad data in traditional SCADA measurements and make state estimation. The main advantages of the test equation method are the opportunity to reduce the dimensionality of the problem and to use the obtained test equations for a priori detection of bad data in measurements.

The paper is concerned with the problem of identification and mitigation of the malicious cyber attacks in the EPS state estimation. To this end, we consider SCADA and WAMS structures, reveal vulnerable вЂњpointsвЂќ, and analyze potential cyber attacks. Special attention is paid to hidden cyber attacks, aimed at distorting the state estimation results.

In this connection, we propose an algorithm for detection and mitigation of cyber intrusions. The algorithm is based on test equations as an additional stage in state estimation. The SCADA data were used to implement the algorithm under simulated cyber attacks. The obtained results showed effectiveness of the algorithm in state estimation.

Methods of hierarchical optimization of hydraulic modes of heat supply systems

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The report focuses on problem arising at the stage of planning of hydraulic modes (GM) heat supply systems (HSS). In practice, this problem is solved by multivariate calculations of flow distribution [1]. The choice of ways to organize the modes entirely rests with the engineer. Automation of these tasks is hampered by a number of complexity factors, such as: large dimension; nonlinearity; discrete part of the variables; multicriteriality etc. For these reasons, at the moment there are no suitable for practical application techniques and software packages for optimizing HSS modes.

The report outlines a new, hierarchical approach to optimizing HSS modes, which allows solving the challenge of the dimensionality of the problem and simplifying the challenge of its multicriteria. The method of hierarchical optimization is to perform the following steps: 1) decomposition of hydraulically coupled HSS to main (MHN) and distribution (DHN) heat networks; 2) search for permissible limits for changing the mode parameters at decomposition points that guarantee the existence of admissible DHN modes; 3) optimization of the MHN regime, taking into account these constraints according to the economic criterion; 4) optimization of DHN modes by technological criteria at the values of the boundary conditions at the input of the DHN, obtained in item 3.

The problems arising here are mathematically reduced to the following classes: 1) problems of non-linear mathematical programming;2) mixed (discrete-continuous) mathematical programming problems with integer variables;3) one- and two-criteria mixed conditional optimization problems with boolean variables;

The constructive methods for solving the above problems are proposed, based on a combination of methods of sequential optimization, method of nonlinear programming, method of internal points [2], dynamic programming [1], method of branches and boundaries [3].

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The Model of Power Shortage Estimation of Electric Power Systems with Energy Storages

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The modern electric power systems with renewable plants contain energy storages. For analysis of generation adequacy of such systems we should take into account random character of generation, processes of accumulation and consumption of electricity from energy storages. We have efficient method based on Monte Carlo simulations for analysis of generation adequacy of traditional electric power systems. For adaptation of the method to modern conditions is necessary to modify the model of power shortage estimation.

Let's consider a scheme of electric power system which contains n nodes and set of system links. The given power system contains energy storages. Let N is number of simulated states of electric power system. Each system state is characterised of set of random values such as of available generating capacity $\bar{x}_i^k$, value of load $\bar{y}_i^k$ in node i, power line capacity $\bar{z}_{ij}^k$ between nodes i and j, $i = 1, \ldots, n$, $j = 1, \ldots, n$, $i \neq j$, $k = 1, \ldots, N$. Storage capacity $\overline{\triangle x}_i$ is given for each node i.

Let x_i is power used at the node i, y_i is the power served at the node i, $\triangle x_i$ is variation of battery power condition at i-th energy storage, z_{ij} is power flow from the node i to the node j, $i = 1, \ldots, n$, $j = 1, \ldots, n$, $i \neq j$. I propose following new problem for power shortage estimation of system state with the number k

$$G \sum_{i=1}^{n} y_i + \sum_{i=1}^{n} \triangle x_i \rightarrow max,$$

$$x_i - y_i - \triangle x_i + \sum_{j=1}^{n} (1 - \alpha_{ji} z_{ji}) z_{ji} - \sum_{j=1}^{n} z_{ij} = 0, \ i = 1, \ldots, n, \ i \neq j,$$

$$0 \leq y_i \leq \bar{y}_i^k, \ 0 \leq x_i \leq \bar{x}_i^k, \ -\triangle x_i^{k-1} \leq \triangle x_i \leq \overline{\triangle x}_i - \triangle x_i^{k-1}, \ i = 1, \ldots, n,$$

$$0 \leq z_{ij} \leq \bar{z}_{ij}^k, \ i = 1, \ldots, n, \ j = 1, \ldots, n, \ i \neq j.$$

Here α_{ij} are given positive coefficients of specific power losses during electric energy transfer from the node i to the node j, $i \neq j$, G is weight number for raising of priority of minimization of power deficit, $G > 1$, $\triangle x_i^{k-1}$ are optimal meanings of variables $\triangle x_i$ of given problem for previous system state with the number $k - 1$, $i = 1, \ldots, n$, $j = 1, \ldots, n$, $k = 1, \ldots, N$. Let's take any number from interval $(0, \ \overline{\triangle x}_i)$ in the capacity of $\triangle x_i^0$, $i = 1, \ldots, n$.

Acknowledgments. This work was supported in part by the grant 16-37-00333-mol-a from RFBR.

Influence of Casual And Active Consumer Behavior On Energy Consumption

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Keywords: mixed integer linear programming, energy consumption, demand response, active consumer.

In this paper, we identify casual consumer as a consumer who can reduce his expenses for energy supply sacrificing his comfortable life conditions. Active consumer tries to find trade-off between satisfactions of his energy demand and endeavor to minimize energy expenses using intelligent technology [1].Simulation of consumer's behavior is important for optimization of energy systems because consumption curve is a part of initial information. Scientists used to assume the consumption curve either to be constant during a year or to be divided on three seasons with peculiar characteristics. This allows faster analyzing exploitation of energy systems due to smaller computational resources required with sufficient accuracy. But, in case of small-scale periods of time, consumers' behavior can impact on total and maximal value of energy demand. In simulation of casual consumer's behavior one should account for ambient temperature variation and peculiarities of the day under consideration. These factors can also influence on behavior of active consumer. In case if energy demand is stochastic value rather than determinate one, then solution of the problem of energy systems operation becomes difficult. This problem concerns operation of centralized systems and single facilities of energy generation. This paper is devoted to solution of the exploitation problem for energy supply systems for active and casual consumers on the basis of demand statistics obtained using Monte-Carlo method. Solution of the operation problem involved technique of mixed integer linear programming.

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Methods and Software for Parameter Optimization of Heat Supply Systems

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This paper presents new methods and software system SOSNA intended for the parameter optimization of multi-circuit heat supply systems. They make it possible to calculate large-scale systems which have a complex structure with any set of nodes, sections, and circuits. The research is based on the theory of hydraulic circuits which provides universal methods of mathematical modeling and optimization of heat, water, oil and gas supply systems.

A new methodological approach to solving the problem of the parameter optimization of the multi-circuit heat supply systems is developed [1]. The approach is based on the multi-level decomposition of the network model, which allows us to proceed from the initial problem to less complex sub-problems of a smaller dimension. New algorithms for numerically solving the parameter optimization problems of multi-circuit heat supply systems are developed [2]: 1) an effective algorithm based on the multi-circuit optimization method, which allows us to consider hierarchical creation of the network model in the course of problem solving; 2) a parallel high-speed algorithm based on the dynamic programming method.

The new methods and algorithms were used in the software system SOSNA (in Russian this abbreviation means "Synthesis of Optimal Systems with Due Regard for the Reliability"). The expandable architecture of the software system allows the construction of a flexible adaptive model for controlling computational process. Its presentation in the form of software components makes it universal and permits the repeated use of these components in various software applications when solving the problems of heat supply systems design.

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[1]This work was supported by grant 16–07–00948 from the Russian Foundation for Basic Research.

Risks in the energy sector: analysis the practices of management (for example, vertically integrated companies)

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Energy security is today one of the most discussed in international economic relations, the Russian political and business community, the regions and all consumers. It is being actively discussed by European governments, leading international organizations. But it should be noted that currently no universal definition of energy security is simply not there. In addition to the lack of such a universal definition, analysis of decisions in this area, one notices the lack of attention to such phenomena as risks in the energy sector, especially in the management of investment projects in the industry.

Risk analysis, in our opinion, a multifactorial phenomenon. First of all, it takes some refinement of the concept of "risk". In the study, the correlation of uncertainty, risk and loss.

Uncertainty	→	Risks	→	Loss
Incompleteness or inaccuracy of information about conditions of realization of the project		Possibility of losses due to the uncertainty		Damage loss in connection with the occurrence of the risk event under uncertainty (loss of funds working time, loss of profits, rising costs, environmental damage, etc.)

Risk assessments should consider individual risk tolerance, which is described by curves of indifference or utility. Therefore, it is recommended to describe the risk of the above three parameters:

$$Risk = \{P * L * Y\}$$

Analysis of project risks is based on risk assessments, which are to identify the magnitude (degree) of risk. But first we need to have an understanding of the risks, i.e. to know their classification, classification of risks in business and system factors influencing the level of risk IE.

Risk analysis is conducted from the point of view:

- sources, causes of this type of risk;
- the likely adverse consequences resulting from a possible implementation of this type of risk;
- specific projected activities, minimize risk to consider.

Risk management is a specific area of management that requires expertise in the field of the theory of an industry, company, insurance, analysis of economic activities of the enterprise, mathematical optimization techniques, economic problems, etc.

World experience shows that there is no one right organizational structure. You need to choose the governance structure that is adequate to the current economic conditions of functioning of the company and allows it to achieve its goals. In any structure, you can focus on decentralization of powers, allowing the managers of lower levels to make decisions. The decentralized structure is recommended if the company has access to dynamic markets, diversified production, competitors and rapidly changing technology. Methods of project risk management in the electric power industry can and should become a means of effective implementation of projects at all levels of government – Federal, regional, and local. It is hoped that the problem of minimizing the risks in the energy sector will be reflected.

Online Parameters Selection for T-S Fuzzy Model based Short-Term Wind Power Forecasting

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The wind power forecasting is challenging problem due to the impacts of wind speed and direction, temperature and pressure which all have uncertain nature. The objective of this contribution is to enhance a wind power short-time forecasting method [1] based on the T-S fuzzy model, which does not rely on a large amount of historical data and can linearize the complex nonlinear process to obtain accurate results. In [1] we proposed method, the main affecting factors are selected by means of the correlation analysis for wind power prediction. In this talk an efficient procedure of online parameters selection is employed. The efficiency of proposed approach is demonstrated comparing with the EMD-SVM methods. The results show that the proposed models can effectively improve the precision of the short-term wind power forecasting. The datasets from China and Ireland are employed.

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Concept Drift Handling for Online Voltage Security Analysis using Random Forest

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Ensemble methods are core machine learning methods employ various learning algorithms in order to achieve better predictive accuracy comparing with individual classifiers. For real world power engineerings applications learning algorithms are supposed to work in dynamic environments. In such situation data continuously generated in the form of a stream. Data stream processing normally employ single scans of the training data and implies strict restrictions on memory and time.Changes caused by dynamic environments (e.g. consumer/load behaviour in power grids) can be categorised into sudden or gradual concept drift subject to appearance of novel classes in a stream and the rate of changing definitions of classes.

The objective of this submission is to employ Proximity Driven Streaming Random Forest (or PDSRF) algorithm [1] to assess and control voltage stability in power system in real time. As a target indicator of system stability when training the PDSRF model we use L-index [2] as an indicator of impeding voltage stability.

Based on this idea, we also suggest modifying a L-Q sensitivity analysis method for reactive power optimization, when the reactive power injections are calculated from the L-index minimization conditions, and keep a system under heavy load conditions away from instability boundaries. In traditional statement, this method requires considerable computational efforts and its application in the real time problems can be complicated.

We suggest supplementing a classical sensitivity analysis method by using PDSRF-based models able вЂњto learnвЂќ to calculate both the global L-index for the security assessment of an entire system, and the required reactive power injections, when determining the place and magnitude of corrective actions. This allow us to apply this methodology in real time. Comparative analysis on various IEEE test schemes against the state of the art methods is fulfilled.

Acknowledgments. This work is funded by the RSF grant No. 14-19-00054.

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OPTIMIZATION IN INVERSE PROBLEMS

Application of optimization methods in solving problems of chemoinformatics

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Abstract. The aspects of solving the chemical reaction mapping problem using the Support Vector Machine method and the problem of quadratic convex minimization are considered.

In this message one of problems of chemoinformatics is considered - obtaining chemical reaction mapping, i.e. search for the correspondence of atoms of the left and right parts of the chemical reaction ([1]).

For a mathematical model it is assumed that the chemical reaction is a sequence of operations on the atoms of the left part of the chemical reaction, as a result of which the right part is obtained. Some basic operations are removing the bond from an atom, inserting a bond from an atom, completely detaching or attaching an atom to another are introduced. As a vector describing the reaction, we will use a vector with components-quantities of significant chemical transformation operations. So, we consider the problem of classifying the mappings of a chemical reaction into two classes — correct and incorrect.

To solve this binary classification problem the Support Vector Machine method was applied, which reduces to the optimization problem with following dual problem:

$$\min\left\{\sum_{i=1}^{l}\lambda_i - 0.5\sum_{s=1}^{l}\sum_{t=1}^{l}\lambda_s y_s \lambda_t y_t \langle x_s, x_t\rangle, 0 \le \lambda_i \le C, i = 1\dots l\right\},$$

where $l > 0$ — number of examples in the training sample, $(x_i, y_i)(i = 1\dots l)$ is an example with a class label $y_i(y_i \in \{-1, 1\})$ — correct or incorrect mapping, λ_i $(i = 1\dots l)$ — a set of dual variables along which one can obtain the vector of the coefficients of the separating hyperplane $(\sum_{i=1}^{l}\lambda_i x_i y_i)$.

To solve this dual problem, an algorithm was applied with the decomposition to the construction of coordinate-wise descent algorithms, proposed in [2].

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The geometry of equilibria of orbital gyrostat

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The motion of stationary gyrostat is considered in the so called restricted formulation of problem [1]. The stationary gyrostat considered to be a rigid body with an isotropic axially symmetric rotor which rotates around its axis of symmetry fixed in a rigid body with a constant anqular velocity. The system moves around an attracting center in central newtonian field of forces. The center of mass of system goes on Keplerian circular orbit.

From equations of system motion which can be obtained with the help of principle of the least operation [2] the new form equations for the determining of the relative equilibria (positions of the rest of gyrostat in orbital coordinate system) of orbital gyrostat was derived [3]. By means of that equations it is studied the geometry of equilibria, the structure of the set of equilibria and bifurcation of the set of equilibria with respect to quantity of moment of momentum of rotor and other parameters of orbital gyrostat.

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[1]This work was partially supported by grant 15–08–06680a from the Russian Foundation for Basic Research and grant for SLSS–8081.2016.9.

Robust Determination of Bubbles Size Distribution based on Image Analysis

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Bubbles detection is a very useful in various critical machine vision [1] application areas such as medical technology, process control, energy and the petroleum industry. The most common type of sensors used are ultrasonic or capacitor based. The application of computer vision methods and algorithms in the detection of bubbles and features extraction of bubbles including diameter, area, and radius provide high efficiency and accuracy for bubbles recognizing system. Especially with different environments, the input image has a diverse and complex image background. This is one of the key issues that need to be studied. In this paper, we present a new and effective method to be used in solving the problem of bubbles detection. This method consists of three main steps: First, image preprocessing improves image quality and details each object in the image, followed by background subtraction image and contour detection. Method is proposed to be used for image segmentation based on Graph cuts algorithm, which is applied specifically to those models which employ a max-flow/min-cut optimization. Hough transform algorithm is applied to the collection of a center point of bubbles or components points, which are presented in a contour of bubbles.The efficiency has been optimized for a continuous update of a list of voting points based on the accumulator size and position of bubbles. The final step is identification and calculation of features. Using a data source provided by ASTI Holdings Pte Ltd (S'pore) [2] to test the proposed method demonstrates the effectiveness of the method when detecting and calculating bubbles that are not too large and do not overlap.

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On A Fast Denoising Technique in Tomography

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Abstract. A fast denoising approach in spectral tomography is investigated. At each iteration step of image reconstruction, total variation is estimated on a longitudinal subdomains of multispectral image. Results of numerical comparative reconstructions are illustrated.

The development of spectral computerized tomography (CT) has made substantial progress in recent years especially in applying the optimization theory methods for reconstruction from few-views CT data [1]. However, acceleration of convergence of iterative algorithms is still an important problem. For approaching this issue, joint image reconstruction and segmentation has already attracted interest of mathematicians [2]. In this work, we are concentrated on a modular approach, where segmentation is implemented and controlled separately to keep the algorithm fast.

During a reconstruction procedure, a subset of voxels of the image under reconstruction along the x-ray penetration domain constitutes a one-dimensional staircase-like strip, or elongated patch, intersecting the image. Total variation (TV) denoising can be applied to this patch in 1-D mode very fast [3]. Considering a certain strip-like subdomain of length M voxels for all K different channels, we obtain an $M \times K$- sized image S. We apply then statistical techniques to segment the image S, detect points of simultaneous change in the rows of image S which indicate the image edges. The segmented patch serves as an input for the next iteration step of the algorithm used for spectral CT in combination with TV regularization.

Acknowledgments. The work is partially supported by the Russian Foundation of Basic Research (grant no. 16-07-00066).

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On signdefiniteness of the forms higher than the second order

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Signdefiniteness of the forms and functions of several variables relates to optimization problems.The forms of more than two variables of the order higher than the 2nd one are of substantial interst. Necessary and sufficient conditions of signdefiniteness for them are known, while including those for the 4th order forms of the three variables $F_4(x_1, x_2, x_3)$. The paper proposes an approach to verification of signdefiniteness, which is based on decomposition $F_4(x_1, x_2, x_3)$ into two quadratic forms of four variables $V_1(x),\ V_2(x)$ $(x \in R^4)$. From the formal viewpoint, form $F_4(x_1, x_2, x_3)$ may be represented as a result of substitution of variable $x_4(x_1, x_2, x_3)$, which satisfies equation $V_1(x_1, x_2, x_3, x_4(x_1, x_2, x_3)) = 0$, into the expression of the quadratic form $V_2(x)$. This is executed by the resultant [1], composed of polynomials $V_1(x),\ V_2(x)$ in case of excluding variable x_4 from them. Signdefiniteness of form $V_2(x)$ on the manifold $V_1(x) = 0$ is according Finsler theorem [2] equivalent to the signedefiniteness of the bundle of the two quadratic forms

$$K(\sigma, x) = V_2(x) - \sigma V_1(x) = x'(M_2 - \sigma M_1)x$$

under some real σ. For the purpose of investigation of signdefiniteness of the bundle of the forms we use the algorithm, which relies on the properties of roots of the characteristic equation $f(\lambda) = det(M_2 - \lambda M_1) = 0$ composed of matrices of quadratic forms, and on relation of signdefiniteness of $K(\sigma, x)$ to simultaneous diagonalization of matrices M_1 and M_2 of quadratic forms [3]. A numerical example is considered.

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Image Colorization

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Keywords: machine learning · neural networks · image processing · generative adversarial networks.

Artificial neural networks is a widely popular class of machine learning algorithms, which has shown exceedingly well results in solving image and text processing tasks [2]. With the advent of deep learning era we can attempt to solve complex inverse problems, in which we learn the parameters of a neural network using the available data. In this paper we propose an approach for solving an image processing problem – colorization of grayscale photographs. We describe the GAN (Generative Adversarial Networks) framework introduced for image generation [1]. Specifically we focus on conditional GANs for solving grayscale-image-to-color-image translation problem [3]. Our approach allows us to obtain plausible color images through an end-to-end framework.

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Modified duality method for solving model crack problem

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Abstract. We consider the duality method based on the use of modified Lagrangian functional for solving a model of elastic problem with a crack. We construct Uzawa method for finding a saddle point, proved the convergence theorems. The results of numerical experiments are given.

Keywords: model problem with a crack, variational inequality, modified Lagrangian functional, Uzawa method.

This paper is devoted to the analysis of model, a wide interest in which is manifested in the recent years. It is a model problem of elastic deformation of the body containing a crack. The formulation of this type of problem can be found in monograph by A M Khludnev [1]. Model proposed here differs from classical approach to the crack problem because it is characterized by the nonlinear boundary conditions on crack faces. Suitable boundary conditions are written as inequalities which provide mutual nonpenetration between crack faces. From the standpoint of mechanics such models are more preferable than the linear classical models.

The modified Lagrangian functional for the first time were developed and investigated for solving the problem of finite-dimensional optimization. Their emergence was related to the fact that classical Lagrangian functionals that are linear functions of the dual variables are not suitable for solving the singular optimization problems. The construction of modified Lagrangian function (functional) actually comprises regularization of dual variables. In last time the Lagrangian multiplier method is successfully applied to the solution of infinite-dimensional variational inequalities in mechanics. Applying a similar scheme for solving the crack problem complicated by the fact that in the neighborhood of the crack faces the regularity of the solution may be arbitrary bad, and the dual problem may be unsoluble. Despite this problem, it is possible to justify the duality scheme to solve the crack problem, as well as equality of duality for the original and dual problems.

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The Regularization Parameter Choice in the Nonclassical Variational Problems Statements of the Inverse Problems

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Keywords: ill conditioned SLAE, variational problems with non-quadratic functional, estimation of the optimal regularization parameter.

The solution of the inverse problems in many cases can be reduced (after the appropriate sampling) to the solution of system of the linear algebraic equations (SLAE) which matrix has a ill conditionality (and it is possible also degeneracy). In these cases for obtaining the steady solution to use a Tikhonov regularization method with classical quadratic functional. The resulting regularizing algorithm is linear (in case of the given parameter of regularization). Unfortunately, the property of linearity does not allow us to find well a solution vector, in which there are abrupt changes in the amplitude of the projections in combination with the intervals at which the projections are almost constant. Such solutions can be called contrast solutions. To overcome this difficulty in this paper, the regularized solution ϕ_α of SLAE is found from the condition of a minimum of the functional:

$$F_\alpha(\phi) = ||\tilde{f} - K\phi||_2^2 + \alpha||\phi||_1.$$

The presence of a functional $||\phi||_1 = \sum_i |\phi_i|$ determines the discontinuous character of the gradient, that generates the known difficulties connected to convergence of iterative procedures. In the literature, an algorithm for minimizing the functional using wavelet functions is proposed. However, the choice of the regularization parameter itself, which has a significant effect on the accuracy of the regularized solutions obtained, has not been solved.

Therefore, in this paper the main attention is paid to the construction of a statistical algorithm for estimating the optimal regularization parameter that minimizes the mean square error of the regularized solution and based on checking the statistical hypotheses about the residual vector. The carried out researches have shown that the proposed algorithm for choosing the regularization parameter allows to estimate with an acceptable accuracy the optimal value of the regularization parameter and can be successfully used for solving practical invers problems.

Data Fitting under Interval Error: Centroids of Feasible Parameter Set

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The classical approach to data fitting is based on stochastic assumptions about the measurement error, and provides estimates that have random nature. In contrast, interval data fitting only assumes the knowledge of deterministic error bounds, and allows obtaining of guaranteed interval estimates for a feasible parameter set (FPS) [1]. Along with interval estimates, in real-life applications, it is often important to choose point estimate in FPS [2].

Generally speaking, all points in FPS are equally possible as we have no information to prefer one or another member of FPS. Nevertheless, in practice some more or less grounded recipes are employed. Most of them recommend to use one or another so called center point or centriod of FPS as a point estimate.

Our work is aimed at the study of properties of estimates constructed as FPS centroids for fitting linear-parameterized dependency $y = x\beta$ ($y \in \mathbb{R},\ x, \beta \in \mathbb{R}^p$) to a set of observations $D_N = \{(x_j, y_j, \varepsilon_j) \mid j = 1, \ldots, N\}$ where inputs x_j are observed without errors while output y_j is measured with the error absolutely bounded by ε_j. The following FPS centroids are considered:

- outer interval estimate center (center of FPS minimum bounding box);
- Chebyshev center (minimax or worst-case estimate);
- L_1-center;
- Oskorbin center (restricted L_1-center);
- center of maximum length diagonal;
- analytic center;
- point minimizing average distance to verticies;
- gravity center;
- Fréchet mean;
- maximum density point for interval outer estimate of FPS.

The study is based on the extensive numerical simulation. For some known p, N and ground truth value β^* we simulate data set D_N where error of y_j has known distribution (uniform, triangle, truncated normal) and compute centroids from the above list. Using centroids as point estimates $\hat{\beta}$ we study the accuracy of estimates (proximity to β^*), their variation and behavior depending on dimension p, observations number N, radius of interval error ε.

The results of simulation study show that in most number of cases analytic center and Oskorbin center give better point estimates than other FPS centroids, i.e. they provide the lowest mean and median distance between $\hat{\beta}$ and β^* as well as the lowest standard deviation and interquartile range.

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Inverse problems for decision principle in media planning

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This article deals with the inverse problems for the optimization models in media planning [1]. The choice of simulation parameters and fundamental numeric indicators based on the recommendations in [2].

Mathematical models include on the one hand the internal parameters on the other hand external parameters. These parameters reflect the environment impact on the decision-making process.On this point the effectiveness and sufficiency of mathematical modeling the inverse problem allow to consider the desired properties of the desired solutions in the model. As a result is, the mathematical model differs from classical optimization problems. The efficiency of these problems requires new techniques for finding solutions. In addition, it is necessary basically new mathematical constructs for the solutions formulation of these problems.

Using the definition of inverse problems, we have a choice, which of two tasks is the direct and which is reversed. Usually simpler and better studied task takes as a direct one.

Advertising specialist has possibilities to advertising positioning and a creative brand strategy developing by using the developed models. If it is necessary, you can plan the advertising budget allocation scheme. In addition, you can calculate the optimal cost of the advertising promotion. In that way there is no need to use an external advertising agencies.

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[1]This work was supported by grant 15–41–04436 from the Russian Foundation for Basic Research.

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Тезисы докладов XVII Байкальской международной школы-семинара «Методы оптимизации и их приложения»

Утверждено к печати Институтом систем энергетики им. Л. А. Мелентьева СО РАН

Подписано к печати 12.07.2017

Бумага офсетная
Формат 70x100 1/16
Печ. л. 13.7
Тираж 100 экз.
Заказ №

Отпечатано полиграфическим участком ИСЭМ СО РАН
664033, Иркутск, ул. Лермонтова, 130

NATIONAL RESEARCH UNIVERSITY
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