On signdefiniteness of the forms higher than the second order

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Signdefiniteness of the forms and functions of several variables relates to optimization problems.The forms of more than two variables of the order higher than the 2nd one are of substantial interst. Necessary and sufficient conditions of signdefiniteness for them are known, while including those for the 4th order forms of the three variables $F_4(x_1, x_2, x_3)$. The paper proposes an approach to verification of signdefiniteness, which is based on decomposition $F_4(x_1, x_2, x_3)$ into two quadratic forms of four variables $V_1(x),\ V_2(x)$ $(x \in R^4)$. From the formal viewpoint, form $F_4(x_1, x_2, x_3)$ may be represented as a result of substitution of variable $x_4(x_1, x_2, x_3)$, which satisfies equation $V_1(x_1, x_2, x_3, x_4(x_1, x_2, x_3)) = 0$, into the expression of the quadratic form $V_2(x)$. This is executed by the resultant [1], composed of polynomials $V_1(x),\ V_2(x)$ in case of excluding variable x_4 from them. Signdefiniteness of form $V_2(x)$ on the manifold $V_1(x) = 0$ is according Finsler theorem [2] equivalent to the signedefiniteness of the bundle of the two quadratic forms

$$K(\sigma, x) = V_2(x) - \sigma V_1(x) = x'(M_2 - \sigma M_1)x$$

under some real σ. For the purpose of investigation of signdefiniteness of the bundle of the forms we use the algorithm, which relies on the properties of roots of the characteristic equation $f(\lambda) = det(M_2 - \lambda M_1) = 0$ composed of matrices of quadratic forms, and on relation of signdefiniteness of $K(\sigma, x)$ to simultaneous diagonalization of matrices M_1 and M_2 of quadratic forms [3]. A numerical example is considered.

References

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2. Kuzmin P.A. Small Oscillations and Stability of Motion. Moscow: Nauka Publ., 1973. 206 p.
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