Decomposition Approach to Nonconvex Quadratic Programming*

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Consider a quadratic programming problem with nonconvex objective function and linear constraints, and reduce it by means of linear transformation to the form

$$\left.\begin{aligned} &\sum_{i=1}^{p} \lambda_i x_i^2 + \sum_{i=1}^{q} \mu_i y_i^2 + x^\intercal l_x + y^\intercal l_y \to \min_{(x,y)} \, , \\ &Ax + By \leqslant d, \qquad \underline{x} \leqslant x \leqslant \overline{x}, \qquad \underline{y} \leqslant y \leqslant \overline{y} \, , \end{aligned}\right\} \tag{1}$$

where $(\lambda, \mu) = (\lambda_1, \ldots, \lambda_p, \mu_1, \ldots, \mu_q)$ stand for eigenvalues of a matrix in the objective function, besides $\lambda < 0$, $\mu \geqslant 0$. For fixed x, consider the subproblem

$$\sum_{i=1}^{q} \mu_i y_i^2 + y^\intercal l_y \to \min_{y}, \qquad Ax + By \leqslant d, \qquad \underline{y} \leqslant y \leqslant \overline{y} \, , \tag{2}$$

that is, obviously, convex. Let $\varphi(x)$ be the optimal value function of (2). It is easy to determine that $\varphi(x)$ is convex function of x. Thus the problem (1) can be represented as

$$\sum_{i=1}^{p} \lambda_i x_i^2 + \varphi(x) + x^\intercal l_x \to \min_{x}, \qquad \underline{x} \leqslant x \leqslant \overline{x} \, . \tag{3}$$

Constructing iterative process in x, procedures for solving (3) can be obtained. For example, d.c. structure of the problem may be used. Decomposition described above is more effective when the dimension of y is significantly larger than the one of x, since the subproblem (2) causes no difficulties for present-day solvers.

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