Approximation Algorithms for Scheduling Problem with Release Times and Delivery Times

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The problem of minimizing the maximum delivery times while scheduling tasks to parallel identical processors is a classical combinatorial optimization problem. In the notation of Graham *et al.* this problem is denoted by $P|r_j, q_j|C_{max}$ and the special case with one machine is denoted $1|r_j, q_j|C_{max}$.

The goal of this paper is to propose approximate IIT (inserted idle time) [1] algorithms for this scheduling problems.

We consider a system of tasks $U = \{u_1 u_2, \ldots, u_n\}$, which is performed on parallel identical processors. Each task u_i must be processed without interruption for $t(u_i)$ time units on any machine, which can process at most one task at time. Each task u_i has a release time $r(u_i)$, when the task is ready for processing and delivery time $q(u_i)$, its delivery begins immediately after processing has been completed.

A schedule for a task set U is the mapping of each task $u_i \in U$ to a start time $\tau(u_i)$. The task u_i has been delivered at time $L(u_i) = \tau(u_i) + t(u_i) + q(u_i)$. Our objective function is to minimize, over all possible schedules, the maximum delivery time

$$C_{\max} = \max\{\tau(u_i) + t(u_i) + q(u_i) | u_i \in U\}.$$

The stated problem is equivalent to that with release times, due dates instead of delivery times and a maximum lateness criterion which is denoted as $P|r_j|L_{max}$.

We propose 2-approximate inserted idle time algorithms for $P|r_j, q_j|C_{max}$ problem. For $1|r_j, q_j|C_{max}$ we propose a new 3/2- approximate algorithm J/IIT, which runs in $O(n \log n)$ time. The algorithm combines the extended Jackson's rule with algorithm, named EDD/IIT (earliest due date/ inserted idle time).

To compare the effectiveness of proposed algorithms we tested random generated problems of up to 500 tasks. The algorithm J/IIT produced a better solution than algorithm EDD in 71 percent of cases. The solution generated with J/IIT algorithm are on average only 8,3 percent away from the optimal value and this deviation is never more than 12 %.

References

1. Grigoreva,N.: Single Machine Inserted Idle Time Scheduling with Release Times and Due Dates. Proc. DOOR2016. Vladivostoc. Russia. Sep.19-23.2016. Ceur-WS. 2016. Vol. 1623, 336–343 (2016)