Generalized Convexity with Operations Max and Min and its Applications in Economics

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Abstract. Concepts of generalized convexity using the idempotent/tropical operation Max (or Min) are introduced, and their applications in the production and consumption theories are elaborated.

We consider continuous nonnegative functions in space R^n_{++} which consists of the origin and of n-dimensional vectors with strictly positive components. We define idempotent/tropical analogues of the superlinear and sublinear functions and find their representations as optimums of functions which are idempotent/tropical analogues of the inner product.

Symbol $\wedge$ denotes a minimum of numbers or a component-wise minimum of vectors, and symbol $\vee$ denotes a maximum. If $\wedge$ is used instead of $+$ in the definition of subadditive function, one comes to the following definition. A function P is called min-subadditive if

$$P(x \wedge y) \leq P(x) \wedge P(y) \forall x, y \in R^n_{++}.$$

In a similar way, with $\vee$ and $\geq$, a max-superadditive function is defined.

Theorem 1. *The following statements are equivalent.*
(1) Function H increases. (2) Function H is min-subadditive. (3) Function H is max-superadditive.

Functions $(l,x) = \min_i l_i x_i$ and $[l,x] = \max_i l_i x_i$ serve as idempotent/tropical analogues of the inner product.

Theorem 2. *If H is an increasing first-degree-positively-homogeneous function, then there exists a set Λ such that*

$$H(x) = \max_{l \in \Lambda}(l,x) = \min_{l \in \Lambda}[l,x], x \in R^n_{++}.$$

We show that the set (the menu) Λ, as well as the functions (l,x), $[l,x]$ and the representations provided by Theorem 2, are useful in studying properties of basic economic objects, such as production functions, utility functions and corresponding models of production, economic growth, consumer behavior, happiness, etc.