Computational technologies for studying set-valued optimization problems

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The problems of unconditional optimization with a set-valued solution, in our opinion, represent an extremal problems class of current importance, but it is not attracting much attention of specialists. This class includes problems that have a set of solutions for which the value of the function coincides with the global extremum. Among the few works on this subject, monograph [1] should be mentioned, there are a classification of such type functions, a number of basic theoretical results, a search algorithm for global minimum and examples of its use. However, the question of finding the set of all solutions at that stage in the development of numerical methods obviously seemed hopeless and is poorly reflected. Close tasks are considered in the theory of incorrectly posed problems, but the main way used to overcome the problem is to introduce new information into the problem. Usually, it is a regularization of functionals, which also does not involve the search for multiple solutions. Methods for interval analysis can also be used to solve the problems of the class of set-valued functions. But, unfortunately, they require analytic expressions of the function, and the dimensions of the problems are very limited. Specialists say that not more than ten variables can be used (see, e.g. [2]), in our opinion, it is critically small.

However, such problems appears in applications quite often. We can mention the underdefined problems of parametric identification, the problem of finding solutions of nonlinear equations systems, the problem of searching for low-potential atomic-molecular clusters, optimal control problems for transferring the system from point to point etc. The report discusses technologies for approximating a set of solutions, based on the use of irregular grids, so-called "cloud approximations". At first stage, stochastic approaches are used to obtain the primary "cloud". Then, using clustering methods, we separate clusters and evaluate it using implemented software. As an application of the developed technologies we present problems of approximating the level set of a function and the problem of finding a global minimum. The results of numerical experiments are presented.

Acknowledgments. This work is partly supported by the Russian Foundation for Basic Research (project N 15-07-03827).

References

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