Projection on ellipsoid with $O(1/t^2)$ convergence rate

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We consider a simple problem of finding a point on the boundary of n-dimensional ellipsoid, which is the closest to some given point x_g outside the ellipsoid with respect to some differentiable distance function $f(x)$, i.e.

$$\min f(x - x_g), \;\; x \in S = \{x : (x - x_0)^T Q(x - x_0) \le 1\},$$

where Q is a positive definite symmetric matrix. This problem statement occurs in some applications, such as obstacle avoidance in robotics or trust region approach in smooth optimization [1].

In recent paper [2] it was shown that for strongly convex sets, the vanilla Frank-Wolfe optimization algorithm gives a rate $O(1/t^2)$, where t is the number of iterations. We simply apply the FW algorithm to our problem, after that the main subproblem we consider is how to compute minimum of a linear function $\nabla f^T x$ on our ellipsoid, where ∇f is the function gradient at the current point.

This task is easy to achieve with the Cholesky decomposition [3] of matrix $Q = L^T L$. We introduce a linear transformation $z = Lx$, after which our ellipsoid corresponds to a (hyper)sphere, while new linear objective function is given by $\nabla f^T L^{-1} z$. The computation of its minimum on $Z = \{z : (z - z_0)^T (z - z_0) \le 1\}$ is then straightforward and given by a closed formula.

We do hope that due to its simplicity the algorithm can find its application in some real-time settings.The author is obliged to G.Sh. Tamasyan (Saint-Petersburg State University) for problem statement.

References

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