Proximal Bundle Method⋆ With Discarding Cutting Planes

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Abstract. A minimization method from a class of bundle methods [1] is proposed for solving nonlinear programming problem. This method is characterized by opportunity of discarding cutting planes. Cutting planes are dropped at the moment, when the model of the convex function have a good approximation quality of the epigraph of the objective function. Convergence of the proposed method is proved.

Let $f(x)$ be a convex function defined in an n-dimensional Euclidian space $\mathrm{R_n}$, $\partial f(x)$ be a subdifferential of the function $f(x)$ at the point $x \in \mathrm{R_n}$. Assume that for any $s(x) \in \partial f(x)$, where $x \in \mathrm{R_n}$, the equality $\|s(x)\| \leq L$ is defined.

Suppose $f^* = \min\{f(x) : x \in \mathrm{R_n}\}$, $X^* = \{x \in \mathrm{R_n} : f(x) = f^*\}$, $X^*(\varepsilon) = \{x \in \mathrm{R_n} : f(x) \leq f^* + \varepsilon\}$, where $\varepsilon > 0$. A method is proposed for finding some point from the set $X^*(\varepsilon)$ with the following input parameters: $\hat{x} \in \mathrm{R_n}, \hat{\delta} > 0, \hat{\mu} > 0, \hat{\theta} \in (0, 1)$.

0. Initialize start parameters $k = 1$, $x_k = \hat{x}$.

1. Assign $i = 1$, $x_{k,i} = x_k$, $\hat{f}_{k,i}(y) = f(x_{k,i}) + \langle s_{k,i}, y - x_{k,i}\rangle$, $s_{k,i} \in \partial f(x_{k,i})$.

2. Find a point $x_{k,i+1} = \mathrm{argmin}\{\hat{f}_{k,i}(y) + \frac{\hat{\mu}}{2}\|y - x_k\|^2 : y \in \mathrm{R_n}\}$, and compute a number $\delta_{k,i} = f(x_k) - [\hat{f}_{k,i}(x_{k,i+1} + \frac{\hat{\mu}}{2}\|x_{k,i+1} - x_k\|^2)]$.

3. If $\delta_{k,i} \leq \hat{\delta}$, then iteration process is stopped, and return the point x_k.

4. If $f(x_{k,i+1}) \leq f(x_k) - \hat{\theta}\delta_{k,i}$, then $x_{k+1} = x_{k,i+1}$, $k := k + 1$, and go to Step 1.

5. Choose $s_{k,i+1} \in \partial f(x_{k,i+1})$, assign $\hat{f}_{k,i+1}(y) = \max\{\hat{f}_{k,i}(y), f(x_{k,i+1}) + \langle s_{k,i+1}, y - x_{k,i+1}\rangle\}$, $i := i + 1$, and go to Step 2.

It is obtained that the proposed method is stopped after finite steps. According to this result the following assertion is proved.

Theorem 1. *The proposed method constructs some point* $x' \in X^*(\varepsilon)$.

References

1. J. Bonnans, J. Gilbert, C. Lemarechal, C. Sagastizabal: Numerical Optimization: Theoretical and Practical Aspects. Universitext, Springer-Verlag, Berlin. (2003).

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