Nonlinear Covering for Global Optimization

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We consider a nonconvex optimization problem:

$$\begin{cases} \text{maximize} & \varphi(x) \\ \text{subject to} & x \in D, \end{cases} \tag{1}$$

where D is a nonempty compact in $\mathscr{R}^n$ and $\varphi : \mathscr{R}^n \to \mathscr{R}$ is a continuous convex function. Problem (1) is called convex maximization (CM) when $\varphi(x)$ is convex and we call it a piecewise convex maximization problem (PCMP) if $\varphi(x)$ is the pointwise minimum of convex functions $\varphi(x) = \min\{f_1(x), \ldots, f_m(x)\}$.

The Lebesgue set of a function f for $\alpha \in R$ is defined like $\mathscr{L}_f(\alpha) = \{x \mid f(x) \leq \alpha\}$.

A feasible point z is the global maximum for (1) if and only if all points of the domain are no better than z in sense of maximization, in other words:

$$D \subset \mathscr{L}_\varphi(\varphi(z)).$$

In order to present the main idea for solving (1) we give a definition along with an abstract result on an equivalence of problems.

Definition 1 *An open subset C satisfying conditions*

$$C \subset \mathscr{L}_\varphi(\varphi(y)) \text{ and } C \neq int(\mathscr{L}_\varphi(\varphi(y)))$$

is called **a covering set at level** $\varphi(y)$.

Proposition 1 *Let y be a feasible point for* (1) *such that $\varphi(y) = max\{\varphi(x) \mid x \in D\} - \delta$ for some $\delta > 0$. Let also C be a covering set at level $\varphi(y)$. Then the following problem is equivalent to* (1)*:*

$$\begin{cases} \textit{maximize} & \varphi(x) \\ \textit{subject to} & x \in D \setminus C. \end{cases} \tag{CC}$$

The main algorithmic feature now looks like

- to cover the feasible set (the domain) by a union of covering sets.
- if the domain is covered by C totally, then stop and the global optimum is found.
- otherwise, solve problem (CC) for an improvement.

Our objective is to construct an "(union of covering sets)" such that

$$D \subset \text{ (union of covering sets)} \subset \mathscr{L}_\varphi(\varphi(z)).$$

Starting with an initial guess of covering sets, a method bootstraps its way up to ever more accurate "sandwich" approximations to answer "the global optimum" or "improvement". What concerns the covering set, the first that comes to mind, is use balls (spherical set) as a simpler nonlinear shape.

Acknowledgements: This research benefited from the support of the FMJH "Program Gaspard Monge for optimization and operations research", and from the support from EDF.