Integer Programming Methods to Polyomino Tiling Problem

Vadim M. Kartak and Aigul I. Fabarisova

M.Akmullah Bashkir State Pedagogical University
Oktyabrskoy Revolutsii st. 3a, 450000, Ufa, Russia
kvmail@mail.ru

Abstract. This paper presents the approach based on integer programming to the problem of polyomino tiling. Two cases are shown: tiling with L-shaped trominoes and tiling with L-shaped tetrominoes. The IP mathematical model and tiling algorithm is described. This problem can be applied to the phased array design where polyomino-shaped subarrays are used to avoid the regularity of antenna structure [1]. Simulations of antenna perfomance show good suppression of sidelobes while the structure fullness is close to maximum.

Keywords: polyomino, integer programming, phased array

We consider the tiling of finite, rectangular region with given polyominoes, without any restriction on their number. Each polyomino can be rotated by 90 degrees and mirror-flipped. So, there is an $N \times N$ element region and infinite number of polyominoes. The problem is to find an optimized layout of polyomino considering two following requirements: to minimize the number of empty spaces and to maximize irregularity i.e. eliminate periodicity of polyominoes layout [2]. Let an $N \times N$ element structure be represented as the set of binary variables $z_{ij} = \{0, 1\}$ where $i = 1, .., N$ and $j = 1, .., N$. Let $z_{ij} = 1$ if it contains the center of polyomino and $z_{ij} = 0$, otherwise. The objective function is to maximize the sum of all variables z_{ij}.

Presented mathematical model and tiling algorithm were implemented in program using Python and IBM ILOG CPLEX solver. Obtained tilings were used to simulate antenna performance. For the case of L-tromino the best peak sidelobe level is -29.15 dB for r = 1.3 with the structure fullness of 99.9%. For the case of L-tetromino the best peak SLL is -25.69 dB for r = 1.3 with the structure fullness of 100%.

References

1. Mailloux R., Santarelli S., Roberts T., Luu D.: Irregular polyomino-shaped subarrays for space-based active arrays. International Journal of Antennas and Propagation. Vol. 2009, P. 9. (2009)
2. Chirikov R., Rocca P., Bagmanov V. et al.: Algorithm for phased antenna array design for satellite communications. Vestnik USATU, Vol. 17 (N. 4(57)), 159–166. (2013)