A 5/6-approximation algorithm for the pseudo-metric TSP-max in an incomplete graph

Aleksey N. Glebov*

Sobolev Institute of Mathematics SB RAS,
4 Acad. Koptyug avenue, 630090 Novosibirsk, Russia

A metric TSP is a well-known variant of the Traveling Salesman Problem where one need to find a Hamiltonian cycle of an extremal weight in a complete weighted graph whose weight function satisfies the triangle inequality. In 1985, Kostochka and Serdyukov [1] proposed an elegant polynomial 5/6-approximation algorithm for the maximization version of metric TSP (abbreviated as metric TSP-max) based on a delicate rearranging the edge set of a maximum-weight cycle cover of a graph. Since then the approximation ratio in [1] was improved several times by various authors. The best known result is a polynomial 7/8-approximation algorithm for the metric TSP-max by Kowalik and Mucha [2].

The purpose of this paper is to extend the 5/6-approximation algorithm of Kostochka and Serdyukov to the case of pseudo-metric TSP-max where the input graph is incomplete but its minimum degree is sufficiently large and the triangle inequality holds for any triangle of the graph. Observe that the weight function of such a graph is not necessarily extendible to the metric weight function of a complete graph. So our pseudo-metric setting of TSP-max is more general when the modification of metric TSP-max where some edges (of a complete graph) are removed (or unavailable). However, our pseudo-metric TSP-max includes such a modification of the metric TSP-max as an important subcase.

As an easy application of our Algorithm, we produce a simple 5/6-approximation algorithm for the maximization version of the metric m-Peripatetic Salesman Problem (metric m-PSP-max). In the metric m-PSP-max one need to find m edge disjoint Hamiltonian cycles of the maximum total weight in a complete weighted graph with triangle inequality. Recently, such a polynomial 5/6-approximation algorithm for the metric m-PSP-max was presented in [3]. However, its description and analysis in [3] are quite complicated.

References

1. Kostochka, A.V., Serdyukov, A.I.: Polynomial Algorithms with Approximation Ratios 3/4 and 5/6 for the Maximization Version of the TSP (in Russian). Upravlyaemye Systemy. Sobolev Institute of Mathematics, Novosibirsk. 26, 55–59 (1985).
2. Kowalik, L., Mucha, M.: Deterministic 7/8-Approximation for the Metric Maximum TSP. Theoretical Computer Science. 410(47-49), 5000–5009 (2009).
3. Glebov, A.N., Gordeeva, A.V.: An Algorithm with Approximation Ratio 5/6 for the Metric Maximum m-PSP // In: Kochetov, Yu. et al (eds.) DOOR-2016. LNCS, 9869, 159–170. Springer, Heidelberg (2016).

* The research is supported by RFBR (projects 15-01-00976 and 15-01-05867).