ON THE RESIDUAL METHOD FOR THE CORRECTION OF CONTRADICTORY PROBLEMS OF CONVEX PROGRAMMING

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Consider the convex programming (CP) problem

$$\min\{f_0(x) \mid x \in X\}, \tag{1}$$

where $X = \{x \mid f(x) \leq 0\}$, $f(x) = [f_1(x), \dots, f_m(x)]$, $f_i(x)$ are convex functions defined on $\mathbb{R}^n$ $(i = 0, 1, \dots, m)$. The CP problems with contradictory constraints ($X = \varnothing$) represent [1] an important class of improper CP problems (ICPP).

The improper problems often appear because of the errors in the initial data, that is connected with the stability of the optimal elements. Similar problems are an object for the theory of ill-posed optimization problems. This fact makes it possible to consider the standard regularization methods of the ill-posed models for the analysis of improper problems. In this report the possibility of the use of the residual method [2] for the approximation of the ICCP is investigated.

In the residual method the following similar problem is considered along with (1):

$$\min\{\|x\|^2 \mid x \in X \cap M_\delta\}, \tag{2}$$

where $M_\delta = \{x \mid f_0(x) \leq \delta\}$, $\delta \in \mathbb{R}^1$, $\delta \geq f^*$, f^* is the optimal value of problem (1). The problem (2) has a unique optimal point x^*_δ, and it is easy to verify, that sequence x^*_δ converge to the normal solution of the problem (1) as $\delta \to f^*$.

If the set X is empty, then the problem (2) is also the ICCP. In this case we register the restrictions of the problem (2) by a penalty function. There are considered the method of the quadratic penalty function and the exact penalty function method. Then instead (2) there are investigated the problem

$$\min_x\{F_\delta(x, r) = \|x\|^2 + \rho\|f^+(x)\|^2 + \rho_0(f_0(x) - \delta)^{+^2}\}, \tag{3}$$

and the problem

$$\min_x\Big\{\Phi_\delta(x, r) = \|x\|^2 + \rho\sum_{i=1}^{m} f_i^+(x) + \rho(f_0(x) - \delta)^+\Big\}, \tag{4}$$

respectively, where $r = [\rho, \rho_0] \in \mathbb{R}^2$, $r > 0$, $\delta \in \mathbb{R}^1$. The both problems (3), (4) have a unique solution for any r, δ and also for $X = \varnothing$. This reason admits to use the functions $F_\delta(x, r)$ and $\Phi_\delta(x, r)$ for the analysis of the ICCP.

In the report specific attention is paid to finding estimates, that characterize the convergence of the solutions (3) and (4) to a approximate solution of the ICCP.

References

1. Eremin, I. I., Mazurov, V. D. and Astaf'ev, N. N. *Improper Problems of Linear and Convex Programming.* Moscow: Nauka, 1983 (in Russian).
2. Vasil'ev F. P. *Methods of Solving Extremal Problems.* Moscow: Nauka, 1981 (in Russian).