THE REFINEMENT OF THE SOLUTIOIN ESTIMATION FOR AN INDIVIDUAL SET COVER PROBLEM

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The set cover problem is considered [1]. The approximation ratio of the algorithm Alg is a value $\rho(Alg)$ such that $c(Alg)/c(Opt) \leq \rho(Alg)$, where $c(Alg)$ is the upper bound for individual problems solutions that is obtained by algorithm Alg, $c(Opt)$ is the wheight of the optimal solution.

Both wheighted and non-wheighted instances of the problem are NP-hard [1]. As it shown in [2], on condition that $P \neq NP$, the greedy algorithm is the asymptotically best algorithm for the problem solution. Let Gr be a cover that is obtained by the greedy algorithm and let Opt be an optimal cover. We have [3] that $c(Gr)/c(Opt) \leq H(m) \leq \ln m + 1$, where $H(m) = \sum_{k=1}^{m} 1/k$. This estimation may be refined for an individual problem: $c(Gr)/c(Opt) \leq H(m')$, where m' is the maximum cardinality of the sets in the optimal cover. Since the estimation generally cannot be improved [4], we may use some parameters of an individual problem to obtain a refinement of the estimation. Thus, for example, because it is computationally hard to obtain m', we may take the maximum cardinality of the given problem sets for m'. Also, it is possible to refine the estimaton for some special cases of the problem.

We propose a proof of the logarithmic estimation for the greedy algorithm that differs from the presented in [3] and allows us to obtain a refinement of the estimation for an individual set cover problem. For the large share of individual problems the refinement is better than the refinement $c(Gr)/c(Opt) \leq \ln m'$.

REFERENCES

1. M. R. Garey, D. S. Johnson *Computers and Intractability.* — W. H. Freeman and Company, 1979.

2. R. Raz, S. Safra *A Sub-constant Error-probability Low-degree Test, and Sub-constant Error-probability PCP Characterization of NP.* — STOC'97 Proceedings of the Twenty-ninth Annual ACM Symposium on Theory of Computing, 1997, pp. 475-484.

3. V. Chvatal *A Greedy Heuristic for The Set-covering Problem.* — Mathematics of Operation Research, V. 4, No. 3, 1979, pp. 233-235.

4. Feige, U. *A Threshhold of* $\ln n$ *for Approximating Set Cover.* — J. ACM 45, 1998, No. 4, pp. 634-652.