DUAL BARRIERS AND REGULARIZATION IN NUMERICAL ANALYSIS OF IMPROPER LINEAR PROGRAMS OF THE FIRST KIND[1]

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We consider the primal linear program

$$\min\{(c,x)\colon Ax=b,\ x\geq 0\} \tag{1}$$

and the dual one

$$\max\{(b,y)\colon A^Ty\leq c\} \tag{2}$$

under the only assumption that the feasible set Y of (2) is nonempty. It means that the program (1) may be either solvable (if its constraints are consistent) or improper of the 1-st kind [1] (if it is not). In the second case the program (1) can be transformed to a solvable one by means of any suitable correction of the right-hand-sides of its constraints.

Following [1] one can put program (1) into parametric family of linear programs like

$$\min\{(c,x)\colon Ax=b-u,\ x\geq 0\} \tag{3}$$

and denote the set of all u which guarantee the consistence of constraints of (3) (and solvability of (3)) by Ω. We determine the optimal correction vector as

$$u_0 := \arg\ \min\{\|u\|\colon u\in\Omega\};$$

where $\|\cdot\|$ is Euclidean norm. It is clear that in a proper case $u_0=0$.

Let X_0 be the optimal set of (3) corresponding to $u=u_0$; $e=(1,\dots,1)$, $A_1,\dots,A_n$ be the columns of the matrix A. To combine the process of correction of (1) with the process of optimization of its criterium we suggest to use the standard logarithmic barrier function for the dual problem (2) supplemented with quadratic regularization term:

$$B(\varepsilon;y)=(b,y)+\varepsilon_1\sum_{i=1}^{n}\ln(c_i-(A_i,y))-\frac{\varepsilon_2}{2}\|y\|^2. \tag{4}$$

Theorem. *If the set Y has at least one inner point then for any pair $\varepsilon=(\varepsilon_1,\ \varepsilon_2)>0$ there exists the unique vector $\hat{y}_\varepsilon$ that belongs to the interior of Y and maximize the function* (4). *More over,*

$$\hat{x}_\varepsilon := \varepsilon_1\mathrm{diag}\Big(c-A^T\hat{y}_\varepsilon\Big)^{-1}e\to X_0,\qquad \varepsilon_2\hat{y}_\varepsilon\to u_0$$

as $\varepsilon=(\varepsilon_1,\ \varepsilon_2)\to+0$.

Note that second order optimization methods may be used for maximization of a smooth concave function like $B(\varepsilon;\cdot)$.

REFERENCES

1. I.I. Eremin, Vl.D. Mazurov, N.N. Astaf'ev. *Improper problems of linear and convex programming.* M.: Nauka, 1983. –336 p. (In Russian)

[1]Report is supported by RFFR (grants 13-01-00210, 13-07-00181) and programs of Presidium UB RAS (grants 12-П-1-1016, 12-С-1-1017/1, 12-П-1-1023 and 12-П-1-1034).