POST-OPTIMAL ANALYSIS OF A VECTOR BOOLEAN PORTFOLIO OPTIMISATION PROBLEM WITH EXTREME OPTIMISM CRITERIA[1]

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We consider s-criterial discrete variant of Markowitz's investment problem [1] with extreme optimism criteria:

$$\max_{x\in X}\ \max_{i\in N_m}\sum_{j\in N_n} e_{ijk}x_j, \quad k\in N_s=\{1,2,\dots,s\}, \tag{1}$$

where e_{ijk} be an expected assessment of efficiency (yield) of measure $k\in N_s$ of investment project j in the situation when the market is in state i; $X\subset \mathbf{E}^n=\{0,1\}^n$ be a set of all admissible investment portfolios $x=(x_1,x_2,\dots,x_n)$, where $x_j=1$ if the project $j\in N_n$ is implemented, and $x_j=0$ otherwise. Pareto set of the investment problem (1) is denoted by $P^s(E)$, where $E=[e_{ijk}]\in\mathbf{R}^{m\times n\times s}$. We define arbitrary Hölder norm l_p, $1\le p\le\infty$, in spaces $\mathbf{R}^n$ and $\mathbf{R}^s$, and the Chebyshev norm l_∞ in the space $\mathbf{R}^m$, i. e. by the norm of a matrix $E\in\mathbf{R}^{m\times n\times s}$ is meant $\|E\|_{p\infty p}=\|(\|E_1\|_{p\infty},\|E_2\|_{p\infty},\dots,\|E_s\|_{p\infty})\|_p$, where $\|E_k\|_{p\infty}=\|(\|e_{1k}\|_p,\|e_{2k}\|_p,\dots,\|e_{mk}\|_p)\|_\infty$, $k\in N_s$. Here $E_k\in\mathbf{R}^{m\times n}$ is k-th cut of matrix E, $e_{ik}=(e_{i1k},e_{i2k},\dots,e_{ink})$ is i-th row of that cut. As usual [2], the stability radius $\rho(m,n,s,p)$ of the problem (1) is defined as $\sup\Xi(p)$ if $\Xi(p)\neq\emptyset$. Otherwise we consider $\rho(m,n,s,p)=0$. Here $\Xi(p)=\{\varepsilon>0:\forall E'\in\Omega(p)\quad (P^s(E+E')\subseteq P^s(E))\}$, $\Omega(p)=\{E'\in\mathbf{R}^{m\times n\times s}:\|E'\|_{p\infty p}<\varepsilon\}$.

Theorem. *For $P^s(E)\neq X$, any $m,n,s\in\mathbf{N}$ and $p\in[1,\infty]$ for the stability radius the following bounds are true*

$$\varphi\le\rho(m,n,s,p)\le(ns)^{1/p}\psi,$$

where

$$\varphi=\min_{x\notin P^s(E)}\ \max_{x'\in X(x,E)}\ \min_{k\in N_s}\frac{f_k(x')-f_k(x)}{\|x'\|_q+\|x\|_q},\quad \psi=\min_{x\notin P^s(E)}\ \max_{x'\in X(x,E)}\ \min_{k\in N_s}\frac{f_k(x')-f_k(x)}{\|x'-x\|_1},$$

$$X(x,E)=\{x'\in P^s(E):f(x')\ge f(x)\quad \&\quad f(x')\neq f(x)\},\quad f(x)=(f_1(x),\dots,f_s(x));$$

$$f_k(x)=\max_{i\in N_m}\sum_{j\in N_n}e_{ijk}x_j,\ k\in N_s;\quad 1/p+1/q=1.$$

REFERENCES

1. H.M. Markowitz *Portfolio selection: efficient diversification of investments.* New Yourk: Willey, 1991, 400 p.
2. V.A. Emelichev, V.V. Korotkov *On stability of a vector Boolean investment problem with Wald's criteria.* — Discrete Math. Appl. — 2012, v.22, No.4, p. 367-381.

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