TWO FAST ALGORITHMS OF THE PROJECTION OF A POINT ONTO THE CANONICAL SIMPLEX[1]

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In this report we consider two fast algorithms of projecting a point $c \in \mathbb{R}^n$ on a standard simplex $\Lambda \subset \mathbb{R}^n$ which is defined by the following conditions:

$$\sum_{i=1}^{n} x_i = 1; \quad x_i \geq 0,\ i \in 1:n.$$

The problem in question is stated as follows:

$$\tfrac{1}{2}\sum_{i=1}^{n}(x_i - c_i)^2 \to \min_{x\in\Lambda}, \tag{1}$$

where $c_1, \ldots, c_n$ are the coordinates of the projected point c. A solution of this problem exists, and it is unique. We denote it by x^*.

A fast algorithm of finding x^* was described in [1]. The idea of the algorithm is based on algebraic analysis of the optimality conditions in Kuhn–Tucker form for the problem (1).

Earlier, the paper [2] also offering a finite algorithm of solving the problem (1) appeared. This algorithm has a geometric nature, which is confirmed in a recent work [3].

In this report we propose an improved version of description and argumentation of the algorithm from [2] and provide the results of numerical experiments comparing two fast algorithms of solving the problem (1). We notice an interesting peculiarity (cf. [4]): when one of the two algorithms takes the maximal working time, the working time of the other one is minimal.

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