A cutting method with updating approximating sets and its combination with other algorithms

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We propose a conditional minimization method which belongs to the class of cutting methods with approximation of the epigraph of the objective function (e. g., [1]). One of its principal features consists in periodically dropping of cutting planes, where there are some advantages from the practical viewpoint.

We solve a minimization problem of a convex function $f(x)$ on a closed convex set $D \subset R_n$. The proposed method for solving the problem consists in the following. Choose a point $v^j \in intE$ for each $j \in J = \{1, \ldots, m\}$, where $E = \{(x, \gamma) \in R_{n+1} : x \in R_n,\ \gamma \geq f(x)\}$, construct a closed convex set $M_0 \subset R_{n+1}$ which contains E, put numbers $\varepsilon_k > 0$, $k \in K = \{0, 1, \ldots\}$, $\varepsilon_k \to 0$, $k \to \infty$, $\overline{\gamma} \leq f^* = \min\{f(x) : x \in D\}$, set $i = 0$, $k = 0$.

1. Find a decision (y_i, γ_i) of the problem

$$\min\{\gamma : (x, \gamma) \in M_i,\ x \in D,\ \gamma \geq \bar{\gamma}\},$$

where $y_i \in R_n$, $\gamma_i \in R_1$

2. If $f(y_i) - \gamma_i > \varepsilon_k$, then put $Q_i = M_i$, $u_i = y_i$. Otherwise choose a closed convex set $Q_i \subset R_{n+1}$ which contains E and a point $x_k \in D$ such that $f(x_k) \leq f(y_i)$, put $\sigma_k = \gamma_i$, increase the value of k by one.

3. For each $j \in J$ choose a point $z_i^j \notin intE$ in an interval $(v^j, (u_i, \gamma_i))$ according to some rule and construct a finite set A_i^j of normalized general support vectors for the set E in the point z_i^j.

4. Put $M_{i+1} = Q_i \bigcap\limits_{j \in J} \{w \in R_{n+1} : \langle a, w - z_i^j \rangle \leq 0\ \forall a \in A_i^j\}$, increase the value of i by one, and go to Step 1.

We prove an optimality theorem for the point y_i. We establish that the method constructs the basic sequence $\{x_k\}$, $k \in K$, with the sequence of auxiliary points y_i, $i \in K$, and we obtain the equality $\lim\limits_{k \in K} f(x_k) = f^*$ for the sequence $\{x_k\}$, $k \in K$.

In case of the strongly convex function $f(x)$ with the parameter μ we obtain $||x_k - x^*|| \leq \sqrt{\varepsilon_k/\mu}$, where x^* is a solution of the problem.

We discuss construction ways of points ε_k and sets M_0, Q_i, A_i^j. We show how due to choosing sets Q_i which differ from M_i, in particular $Q_i = R_{n+1}$, the method allows to update approximating sets M_{i+1} by dropping accumulated any number of cutting planes. Note that in case of $x_k \neq y_{i_k}$ choosing condition of the point x_k allows to combine this method with other algorithms saving its convergence, and to use parallel computings for finding x_k.

REFERENCES

1. V.P. Bulatov *Embedding methods in optimization problems.* Novosibirsk: Nauka, 1977, 161 p.