CONVERGENCE OF THE EXTRAGRADIENT METHOD IN A FINITE NUMBER OF ITERATIONS [1]

A.V. Zykina, N.V. Melenchuk

Omsk State Technical University, Omsk, Russian
e-mail: avzykina.mail.ru

The article deals with double-step extragradient method for solving variational inequalities. It is reported that convergence in a finite number of steps uder the severity conditions.

To solve the variational inequality it is means that to find a vector $z^* \in \Omega$ which satisfies the following conditions:

$$\langle H(z^*), z - z^* \rangle \geq 0, \quad \forall\, z \in \Omega, \tag{1}$$

Where $H : R^n \to R^n$, Ω – convex closed set, $\Omega \subset R^n$, $z^* \in \Omega^*$ – set of solutions for the variational inequality, $\Omega^* \subset \Omega$.

The convergence of the two-step extragradient method for solving $z^* \in \Omega^*$ variational inequalities (1) with monotone operator under the Lipschitz condition with a constant $L > 0$, is provided by the value of the step α which satisfies the following conditions $0 < \alpha < \frac{1}{\sqrt{3}L}$ [1].

Let's consider the additional condition for continuous monotone operator $H(z)$ which is a condition of sharpness. It is means [2] : The condition

$$\langle H(z), z - z^*(z) \rangle \geq \gamma \|z - z^*(z)\|, \quad \forall\, z \in \Omega, \quad z^*(z) = P_{\Omega^*}(z). \tag{2}$$

is performed for the variational inequality, where $\gamma > 0$.
More strict condition $\langle H(z), z - z^* \rangle \geq \gamma \|z - z^*\|$ of sharpness, which assumes the exist the unique solution $z^* \in \Omega$, that was introduced in [3]. The sharpness condition (2) for variational inequalities (1) with potential mapping $H(z) = \nabla f(z)$ is a known condition of sharp minimum $f(z) - f(z^*(z)) \geq \gamma \|z - z^*(z)\|$, $\forall\, z \in \Omega$, for the corresponding aim for convex optimization with convex closed set of solutions $\Omega^* \subset \Omega$ [4].

The convergence of the sequence $\{z^k\}$ for a variational inequality (1) under the condition of sharpness is showed. The sequence $\{z^k\}$ is defined by the recurrence relations of the two-step extragradient method and is converged to the solution $z^* \in \Omega^*$ of the variational inequality (1) for the finite number of iterations.

A similar result is obtained in the one-step method extragradient [3] under the condition of sharpness.

ЛИТЕРАТУРА

1. A.V. Zykina, N.V. Melenchuk *A two-step extragradient method for variational inequalities.* – Russian Mathematics. – 2010, v. 54, № 9, p. 71-73.
2. I.V. Konnov *Combined relaxation methods for variational inequalities.* Berlin: Springer-Verlag, 2001, 184 p.
3. A.S. Antipin *Gradient and extragradient approaches in bilinear equilibrium programming.* M.: CC RAS, 2001, 69 p.
4. B.T. Poljak *Introduction to optimization. Ed. 2-e, Rev. and additional* M.: LENARD, 2014, 393 p.

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