1d AND 2d ELLIPSOIDS IN CONVEX PROGRAMMING

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In this report, we discuss topic of „building of an algorithm, which would be practically no less efficient than r-algorithm and would be as substantiated as ellipsoid method“, initiated by N.Z. Shor in 1982. Ellipsoid method uses space dilation operator $R_\alpha(\xi) = I + (\alpha - 1)\xi\xi^T$. With this, a minimal volume ellipsoid, containing half-sphere in n-dimensional Euclidean space, is trasformed into a new sphere in one space dilation(1d-ellipsoid).

Lemma [1]. *Let B_k be nondegenerate $n \times n$-matrix, with $\|B_k^{-1}(x_k - x^*)\| \le r$; g_1 and g_2 – n-dimensional vectors, complying with $(x_k - x^*, g_1) \ge 0$ and $(x_k - x^*, g_2) \ge 0$. If the condition $-\|B_k^T g_1\|\|B_k^T g_2\| < (B_k^T g_1, B_k^T g_2) < 0$ is satisfied and matrix B is recalculated according to the rule*

$$B_{k+1} = B_k R_{\beta_1}\left(\frac{\xi - \eta}{\|\xi - \eta\|}\right) R_{\beta_2}\left(\frac{\xi + \eta}{\|\xi + \eta\|}\right), \quad \xi = \frac{B_k^T g_1}{\|B_k^T g_1\|}, \ \eta = \frac{B_k^T g_2}{\|B_k^T g_2\|},$$

where $\beta_1 = \sqrt{1 + (\xi, \eta)}$ и $\beta_2 = \sqrt{1 - (\xi, \eta)}$, then matrix B_{k+1} is nondegenerate and has the following properties:

(i) $\|B_{k+1}^{-1}(x_k - x^*)\| \le r$; (ii) $\det(B_{k+1}) = \det B_k \sqrt{1 - (\xi, \eta)^2}$; (iii) $(B_{k+1}^T g_1, B_{k+1}^T g_2) = 0$.

Lemma has the following interpretation: Property (i) assures point x^* location withing the next 2d-ellipsoid, and property (ii) means its volume decrease in comparison with that of previous ellipsoid. Property (iii) assures use of anti-ravine method like one used in r-algorithms. This means that subgradients with obtuse angle in current variable space become orthogonal in transformed space, that allows to improve level surfaces of ravine function. Coefficients of space dilation in direction of difference of normalized subgradients and in direction of sum of normalized subgradients are defined by angle between subgradients. The more obtuse the angle, the more space dilation coefficient in the direction of difference of two normalized subgradients.

2d-ellipsoid can be used for construction of accelerated ellipsoid methods for wide class of problems: convex programming problems, finding saddle points of concave-convex functions, special cases of solution of variational inequalties, special classes of problems of linear and non-linear complimentarity. For these methods, one can assure speed of convergence close to one of r-algorithms. This is confirmed by subgradient methods with space transformation for finding the minimum point of a convex function with a priori knowledge of function value at the minimum point [2]. They proved to be effective while working with ravine functions.

REFERENCES

1. P.I. Stetsyuk *r-Algorithms and ellipsoids.* – Cybernetics and System Analysis, 1996, Vol. 32, No. 1, pp. 93–110.
2. P.I. Stetsyuk *Ellipsoid methods and r-algorithms.* – Chisinau: Evrika, 2014, 488 p. (in Russian)