LOCAL SEARCH PROCEDURES FOR POLYMATRIX GAMES OF THREE PLAYERS[1]

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The problem of numerical finding of a Nash equilibrium in a 3-player polymatrix game is considered, where the payoff function of each player is the sum of two bilinear terms [1]. Such a game can be completely described by six matrices, therefore, we will call it a hexamatrix game. As well known, various economical conflicts on the oligopolistic market with three players can be modeled by means of a hexamatrix game.

In order to find a Nash equilibrium point in a hexamatrix game $\Gamma = \Gamma(A_1, A_2, B_1, B_2, C_1, C_2)$ an optimization approach is used. The approach is based on the equivalence theorem for the game and a special mathematical optimization problem with a bilinear structure in the goal function ($\sigma := (x, y, z, \alpha, \beta, \gamma)$) [1]:

$$\left.\begin{array}{c} \Phi(\sigma) \overset{\triangle}{=} \langle x, A_1y + A_2z\rangle + \langle y, B_1x + B_2z\rangle + \langle z, C_1x + C_2y\rangle - \alpha - \beta - \gamma \uparrow \max\limits_{\sigma}, \\ \sigma \in D \overset{\triangle}{=} \{(x, y, z, \alpha, \beta, \gamma) \in I\!R^{m+n+l+3} \mid x \in S_m, \;\; y \in S_n, \;\; z \in S_l, \\ A_1y + A_2z \leq \alpha e_m, \quad B_1x + B_2z \leq \beta e_n, \quad C_1x + C_2y \leq \gamma e_l\}, \end{array}\right\} \qquad (\mathcal{P})$$

where $S_p = \{u = (u_1, \ldots, u_p)^T \in I\!R^p \mid u_i \geq 0, \;\; \sum\limits_{i=1}^{p} u_i = 1\}, \; e_p = (1, 1, ..., 1) \in I\!R^p, \; p = m, n, l.$ Components (x^*, y^*, z^*) of a global solution to problem $(\mathcal{P})$ are Nash equilibrium point, and $(\alpha_*, \beta_*, \gamma_*)$ are payoffs of players 1, 2, and 3, respectively, of the game Γ [1].

We propose to solve problem $(\mathcal{P})$ by means of the Global Search Theory in nonconvex problems with (d.c.) functions of A.D. Alexandrov [2]. According to the Theory one of the basic element of a global search is a local search, which takes into account the structure of the problem under scrutiny.

To implement a local search in problem $(\mathcal{P})$, let us apply the idea, first, of splitting variables in several groups, and, after that, of consecutive solving of partial LP problems with respect to groups of variables [2]. This idea has previously demonstrated its efficiency in problems with a bilinear structure. By means of various splitting ways of variables, 12 variants of local search procedures for problem $(\mathcal{P})$ were developed. The convergence of these procedures is investigated, practical stopping criteria are proposed, and computational testing of the local search in random generated hexamatrix games is implemented.

REFERENCES

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2. A.S. Strekalovsky, A.V. Orlov *Bimatrix Games and Bilinear Programming.* FizMatLit, Moscow, 2007, 224 p. (in Russian).

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