POLYNOMIAL TIME APPROXIMATION SCHEMES FOR SOME GENERALIZATIONS OF EUCLIDEAN TRAVELING SALESMAN PROBLEM

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In the Traveling Salesman Problem ("TSP"), we are given n nodes and a distance $d_{i,j}$ for each pair i, j of distinct nodes. Our aim is to find a closed path that visits each node exactly once and incurs the least cost, which is the sum of the distances along the path.

Euclidean TSP is known to be NP-hard [3]. For metric TSP N. Christofides has designed an approximation algorithm that runs in polynomial time and for every instance of problem computes a tour of cost at most $\frac{3}{2}$ times the optimum [2]. The most important result for Euclidean TSP in fixed dimensions is polynomial time approximation scheme which has been developed by S.Arora [1].

A PTAS or Polynomial Time Approximation Scheme is a polynomial time algorithm or a family of such algorithms that, for each fixed $c > 1$, can approximate the problem within a factor $(1 + \frac{1}{c})$. The running time could depend upon c, but for each fixed c there has to be polynomial in the input size.

In this paper we present generalization of polynomial time approximation scheme for Euclidean problem about two salesmen. In this problem in the plane there are n nodes and for each pair i, j of distinct nodes, there is a distance $d_{i,j}$. The goal is to construct two closed paths without common vertices that visit each node and incur the least cost.

The main idea in the algorithm of S.Arora [1] is to perform a recursive geometric partitioning of the plane with nodes. Arora proved the existence of $(1 + \frac{1}{c})$-approximate tour that crosses each line of the partition $O(c)$ times. Such a tour can be found using dynamic programming that runs in polynomial time.

In PTAS for Euclidean problem about two salesmen the parts of salesman paths have to be painted in two colors on the each step of dynamic programming to find optimal combination. It is the most important difference from Arora's algorithm. We also give estimation of running time for our generalization of polynomial time approximation scheme.

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