EXTRAGRADIENT METHODS WITH NUMBER OF ADDITIONAL STEPS [1]

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Variational inequalities are very important class of optimization problems. There are number of methods which solve them and among them gradient methods. Extragradient methods are built on basis of gradient methods, but converge with much weaker conditions on operator. Earlier there were built extragradient methods with two additional steps (see [1-2]), here generalized approach for n additional steps.

Let's specify extragradient method with n additional steps, which solves variational inequality $\langle H(z^*), z - z^*\rangle \geq 0,\ z^*, z \in \Omega$, as follows

$$\begin{array}{l} z_1^k = P_\Omega(z^k - \alpha H(z^k)), \quad z_2^k = P_\Omega(z_1^k - \alpha H(z_1^k)), \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\ z_n^k = P_\Omega(z_{n-1}^k - \alpha H(z_{n-1}^k)), \quad z^{k+1} = P_\Omega(z^k - \alpha H(z_n^k)), \end{array} \qquad (1)$$

where $H : R^N \to R^N$, Ω – closed convex set, $\Omega \subset R^N$, $z^* \in \Omega^*$ – set of solutions for variational inequality, $\Omega^* \subset \Omega$, $\alpha > 0$ – parameter, P_Ω – projection operator on set Ω, z^k – known approximation of solution (or beginning point). As it seen from recurrent relations, main difference from standard extragradient method is in doing n additional steps and only with direction in last point z_n^k takes step from starting point z^k. The aim of this method is to find some point z^* from the set Ω^* of all variational inequality solutions.

Convergence of proposed iterative process (1) proved in next theorem.

Theorem. *Let's assume that variational inequality meets following conditions:*
а) Ω *– closed convex set;*
б) $H(z)$ *– monotone operator:* $\langle H(z) - H(v), z - v\rangle \geq 0,\ \forall z, v \in \Omega$, *which meets Lipchitz condition:* $\|H(z) - H(v)\| \leq L\|z - v\|,\ \forall z, v \in \Omega$;
в) set of solutions Ω^* *for variational inequality on* Ω *is not empty;*
г) $0 < \alpha < \frac{1}{\sqrt{2^{n-1}+1}L}$.
Than sequence $\{z^k\}$, *defined by recurrent relations (1), converge to some* $z^* \in \Omega^*$ *– solution of variational inequality.*

LITERATURE

1. A.V. Zykina, N.V. Melenchuk *A two-step extragradient method for variational inequalities.* – Russian Mathematics. – 2010, v. 54, № 9, p. 71-73.
2. A.V. Zykina, N.V. Melenchuk *A doublestep extragradient method for solving a problem of the management of resources.* – Automatic Control and Computer Sciences. – 2011, v. 45, № 7, p. 452-459.

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