AN APPROXIMATE ALGORITHM FOR SOLVING THE MAXIMUM CLIQUE PROBLEM

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Suppose a simple undirected graph $G(V, E)$ is given. It is required to find a maximum subset of vertices C, every pair of vertices connected by an edge in G [1]. This problem can be represented as a continuous one:

$$(P_\gamma) \quad \begin{cases} < x, A_\gamma \, x > \downarrow min \\ \sum\limits_{i=1}^{n} x_i = 1 \\ x_i \geq 0, i = \overline{1, n} \end{cases}$$

Here A_γ - regularized adjacency matrix of the complement graph $\overline{G}$: $A_\gamma = A_{\overline{G}} + \gamma I_n$, I_n — identity matrix , $0 < \gamma < 1$.

Theorem 1. *C is a maximum clique if and only if the point $z_i = \frac{1}{K}$, $i \in C$, $z_i = 0$, $i \notin C$, $K = |C|$ is a solution of (P_γ).*

Consequence. *If $z \in Sol(P_\gamma)$, then $C = \{i : z_i > 0\}$ - maximum clique dimension K, where K - the number of positive components of z.*

For approximate solving of this problem we suggest a method, consisting of a local search procedure and evaluation of the found point. To find a local minimum point proposed a local search algorithm, which allows to search a strict local minimum from any feasible point for finite steps [2]. Evaluation of the found point based on the following statement.

Theorem 2. *$K \leq \frac{\gamma}{f(z)}$, where $f(x) =< x, A_\gamma \, x >$, $z \in Sol(P_\gamma)$ with $\gamma > 1$.*

Algorithm is tested on problems from the library DTMACS.

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