# LOWER BOUND PROCEDURE IN THE BRANCH AND BOUND METHOD FOR THE SCHEDULING PROBLEM WITH RESTRICTED RESOURCES[1]

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We consider the resource constrained project scheduling problem with precedence and resource constraints (RCPSP). We are given a set of activities $J = \{1, ..., n\}$. The partial order on the set of activities is given by directed acyclic graph $G = (V, E)$. Each activity $j \in E$ is characterized by its processing time $p_j \in Z^+$ and the resource requirement for $r_{jk}(\tau)$ in resource type $k$ on the time interval $[\tau - 1, \tau),\ \tau = 1, ..., p_j$. For each constrained resource type $k$ are known its capacity $R_t^k$ during the time interval $[t-1, t)$. All resources are renewable. Activities preemptions are not allowed. The objective is to compute the schedule $S = \{s_j\}$ that meets all resource and precedence constraints and minimizes the makespan $C_{\max}(S)$. A formal setting of the RCPSP problem is the following:

$$C_{\max}(S) = \max_{j \in E}(s_j + p_j) \ \longrightarrow \min_{s_j} \tag{1}$$

Subject to:

$$s_i + p_i \leq s_j, \quad i \in Pred(j),\ j \in E; \tag{2}$$

$$\sum_{j \in U(t)} r_{jk}(t - s_j) \leq R_t^k, \quad k \in M,\ t = 1, ..., T_k; \tag{3}$$

$$s_j \in \mathbf{Z}^+, \quad j \in E, \tag{4}$$

where the set $Pred(j)$ is the set of immediate predecessor for activity $j \in U$, and $U(t) = \{j \mid s_j < t \leq s_j + p_j\}$ is the set of the activities which are being processed at the time instant $[t-1, t)$ at the schedule $S$.

The considered problem is NP-hard. We propose to use branch and bound method to solve it. Important role in its implementation is an algorithm for computing the lower bound. As the lower bound we use the solution of the relaxed problem where all constrained resources being accumulative. To solve this problem we use a fast relaxed asymptotically exact algorithm [1], its processing times depends on the number of activities $n$ as a function of order $n\ logn$, and the absolute error tending to zero for increasing number of activities. We present results of numerical experiments illustrating quality characteristics of the lower bound obtained by using this algorithm, the test instances for which were taken from the library of test problems PSPLIB.

## REFERENCES

1. E.Kh.Gimadi *On some mathematical models and methods of planning of large-scale projects // Models and methods of optimization: Trudy Akademii Nauk SSSR, Siberian branch. Institut matematiki. Novosibirsk: Nauka, 1988. Vol. 10, P. 89–115 (in Russian)*

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