EFFICIENT ALGORITHMS WITH PERFORMANCE GUARANTEES FOR SOME PROBLEMS OF FINDING SEVERAL CLIQUES IN A WEIGHTED COMPLETE UNDIRECTED GRAPH[1]

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Consider the following m-**Weighted Clique Problem (m-WCP).**

Input: a weighted complete undirected graph $G = (V; E; a; c)$, where $a\colon V \to \mathbb{R}$, $c\colon E \to \mathbb{R}$, and positive integers $L_1, \ldots, L_m$ such that $\sum_{i=1}^{m} L_i \leqslant n$.

Output: a family of disjoint cliques in the graph G of orders $L_1, \ldots, L_m$ with the smallest total weight of vertices and edges contained in these cliques.

We show that the problem is NP-hard in a strong sense and inapproximable in the general case and in some of its important special cases. We propose an approximation algorithm with time complexity $O(n^{m+2} \log n)$ for the case when the weights of vertices of the graph G are nonnegative and the weights of edges either satisfy the triangle inequality or are squared pairwise distances between some points of the Euclidean space.

The idea of approximation for m-WCP is based on the use of the exact solution of the special (auxiliary) problem of finding m disjoint stars of corresponding orders. It is shown that the exact solution of this problem can be find in time $O(n^{m+2} \log n)$ using the transportation algorithm. This algorithm is polynomial if m is fixed. The proposed approximation algorithm for the problem m-WSP has the same time complexity. It is proved that for the Metric m-WCP we have the approximation guarantee

$$2\Big(1 - \frac{\sum_{i=1}^{m} S(B_i^*)}{\sum_{i=1}^{m} L_i S(B_i^*)}\Big),$$

where $S(B_i^*)$ is the total weight of the vertices and the edges in the i-th star of the auxiliary problem, $i = 1, \ldots, m$. For the Quadratic Euclidean m-WCP the approximaton ratio 2 is proved.

The present paper generalizes and deepens the results obtained in [1]. We hope that the proposed approach can be extended for other actual subclasses of m-WCP.

REFERENCES

1. I.I. Ereminy, E.Kh. Gimadi, A.V. Kel'manov, A.V. Pyatkin, and M.Yu. Khachai. 2-Approximation Algorithm for Finding a Clique with Minimum Weight of Vertices and Edges // Proceedings of the Steklov Institute of Mathematics, 2014, Vol. 284, Suppl. 1, pp. S1-S9.

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