THE OPTIMAL SEGMENTATION OF GRAPH

V.V. Bykova

Siberian Federal University, Krasnoyarsk
email: bykvalen@mail.ru

We investigate the following problem which is associated with segmentation graph.

SEGMENTATION GRAPH PROBLEM (or briefly SGP)

Instance: A connected graph $G = (V, E)$, $|V| \geq 2$, $|E| \geq 1$; a non-negative integer $K \leq |V|$; a positive integer $L \leq |E|$.

Question: Is there a set $B \subseteq V$ of cardinality K, which segmentation graph $G = (V, E)$ on the set B generates a set of segments $\mathfrak{R} = \{G_1, G_2, \dots, G_p\}$ and $w(G) = \max\{|E_i| : 1 \leq i \leq p\}$ is the maximum (by number of edges) size of segment $G_i = (V_i, E_i) \in \mathfrak{R}$?

Under segmentation graph $G = (V, E)$ on the set $B \subseteq V$ we understand a partition of the set of edges of G, that two edges belong to the same segment of G_i, if and only if in the graph G_i exists (a, b)-path, that includes both of these edges and no contains vertices belonging to the B, except possibly vertices a and b. The set of vertices V_i of segment $G_i = (V_i, E_i)$ comprises end vertices of edges belonging to E_i.

Similar formulations of the SGP studied previously in [1, 2] for the design of trunk pipeline networks. It is known that such problems are NP-hard.

In this paper, we continued to study the SGP. We offer operation segmentation graph $G = (V, E)$ on the set $B \subseteq V$, which indicates constructively as produce different segmentation. We showed that a set of segments $\mathfrak{R} = \{G_1, G_2, \dots, G_p\}$ connected graph $G = (V, E)$, where $|V| \geq 2$, $|E| \geq 1$, uniquely determined by $B \subseteq V$. For a fixed B the set $\mathfrak{R}$ can be constructed in time $O(|V| + |E|)$. We have proved the properties of segments that show design features of admissible and optimal solution. Presented results can be used to develop algorithms for solving SGP.

REFERENCES

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