Methods of the solution of railway scheduling problems [1]

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We consider a variant of the freight railcar flow problem. In this problem, we need 1) to choose a set of transportation demands between stations in a railroad network, and 2) to fulfill these demands by appropriately routing the set of available railcars, while maximizing the total profit. We formulate this problem as a multi-commodity flow problem in a large space-time graph. Three approaches are proposed to solve the Linear Programming relaxation of this formulation: direct solution by an LP solver, a column generation approach based on the path reformulation, and a "column generation for extended formulations" approach. In the latter, the multi-commodity flow formulation is solved iteratively by dynamic generation of arc flow variables. Three approaches have been tested on a set of real-life instances provided by one of the largest freight rail transportation companies in Russia. Instances with up to 10 millions of arc flow variables were solved within minutes of computational time.

This model has a very large size, and even solving its Linear Programming (LP) relaxation using modern commercial solvers can take hours of computation time for real-life instances. However, a solution of this LP relaxation allows one to obtain a very tight dual bound for the objective function value. This fact has been also noticed for similar models considered in. Therefore, we concentrate on solving the LP relaxation of this formulation, leaving the problem of obtaining an integer solution out of the scope of the paper. In practice, rounding a fractional solution in a straightforward way allows one to obtain an integer solution with very small gap. To solve the LP relaxation faster, we devise two variants of the column generation procedure, where columns represent railcar routes or flows of the railcars of the same type. The first variant we tried is the classic Dantzig-Wolfe approach. In the second variant, called "column generation for extended formulations" in, columns are disaggregated into individual arc flow variables when added to the master. Thus, the master problem is equivalent to the original multi-commodity flow model, but its variables are generated dynamically. On almost all real-life instances provided by the company, either the first or the second variant of the column generation approach significantly outperformed the solution by a solver of the LP relaxation of the original multi-commodity model, preprocessed by a problem-specific procedure.

REFERENCES

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