Saddle-point methods for solving therminal control problems with phase constraints

Anatoly Antipin

Federal Research Center "Computer Science and Control" of RAS, Dorodnicyn Computing Centre, Vavilov st. 40, 119333 Moscow, Russia

We consider a problem of terminal control with phase constraints and the boundary value problem

$$x_1^*, x(t), u^*(t)) \in \operatorname{Argmin}\{\langle\varphi_1(x_1)\rangle \mid G_1x_1 \le g_1,\ x_1 \in X_1 \subseteq R^n, \tag{1}$$

$$\frac{d}{dt}x(t) = D(t)x(t) + B(t)u(t),\ t_0 \le t \le t_1, \tag{2}$$

$$x(t_0) = x_0 \in R^n, x^*(t_1) = x_1^* \in X_1 \subset R^n, \tag{3}$$

$$G(t)x(t) \le g(t),\ x(\cdot) \in AC^n[t_0, t_1],\ u(t) \in U\},$$

$$u(\cdot) \in U = \{u(\cdot) \in L_2^r[t_0, t_1] \mid \|u(\cdot)\|_{L_2} \le \text{const}\}. \tag{4}$$

Here $D(t), B(t)$ − $n \times n, n \times r$ are matrix functions, continuously depending on time, G_1 −$m \times n$, $(m \le n)$ is a fixed matrix, g_1, x_0 are given vectors. The controls $u(\cdot)$ are elements of the space $L_2^r[t_0, t_1]$. U is a convex closed set. We introduce the linearized Lagrange function. Using the Lagrange function, we can formulate sufficient saddle-saddle conditions for the extremum for the problem of terminal control with phase constraints and the boundary value problem in the form convex programming.

$$\frac{d}{dt}x^*(t) = D(t)x^*(t) + B(t)u^*(t),\ \ x^*(t_0) = x_0, \tag{1}$$

$$p_1^* = \pi_+(p_1^* + \alpha(G_1x_1^* - g_1)), \tag{2}$$

$$\eta^*(t) = \pi_+(\eta^*(t) + \alpha(G(t)x^*(t) - g(t))), \tag{3}$$

$$\frac{d}{dt}\psi^*(t) + D^{\mathrm{T}}(t)\psi^*(t) + G^{\mathrm{T}}(t)\eta^*(t) = 0,\ \ \psi_1^* = \nabla\varphi_1(x_1^*) + G_1^{\mathrm{T}}p_1^*, \tag{4}$$

$$u^*(t) = \pi_U(u^*(t) - \alpha B^{\mathrm{T}}(t)\psi^*(t)), \tag{5}$$

where $\pi_+(\cdot), \pi_+(\cdot), \pi_U(\cdot)$ − projetion operators, respectively, onto the positive orthant R_+^m, onto the positive orthant $\Psi_+^n[t_0, t_1]$, $\alpha > 0$, and onto the set of controls U.

Using sufficient conditions, iterative saddle-point methods can be formulated. These methods converge in all components of the solution, namely: convergence in controls is weak, convergence in phase and conjugate trajectories is strong (in the norm of space). Convergence in terminal variables is also strong.

References

[1] Antipin A.S., Khoroshilova E.V.: "Linear programming and dynamics" Ural Mathematical Journal. Vol.1, No.1, 3–18 (2015).