On a linear control problem under interference with a payoff depending on the modulus of a linear function and an integral*

Igor' V. Izmest'ev and Viktor I. Ukhobotov

Chelyabinsk State University, Chelyabinsk, Russian Federation
j748e8@gmail.com, ukh@csu.ru

We consider a linear control problem under the action of an interference

$$\dot{x} = A(t)x + \phi B(t)\xi + \eta, \quad x(t_0) = x_0; \quad x \in \mathbb{R}^m, \quad t \leq p \ .$$

Here, p is given end time of the control process; t_0 is given initial time; $\phi \in \mathbb{R}$ and $\xi \in M \subset \mathbb{R}^s$ are control; set M is connected, compact and symmetric with respect to the origin in $\mathbb{R}^s$; interference η belongs to connected compact $Q \in \mathbb{R}^m$; $A(t)$ and $B(t)$ are continuous for $t_0 \leq t \leq p$ matrices.

Admissible control are non–negative function $\phi(\cdot) \in L_q[t_0, p]$ and arbitrary function $\xi : [t_0, p] \times \mathbb{R}^m \to M$. Interference realizes as arbitrary function $\eta : [t_0, p] \times \mathbb{R}^m \to Q$. This definition of the control arises in control problems for mechanical systems of variable composition [1]. For example, the law of variation of a reaction mass is defined as a function of time, and the control affects the direction of relative velocity in which the mass is separated.

A quality index of control is value

$$G(|\langle \psi, x(p) \rangle - C|) + \int_{t_0}^{p} \phi^q(r)dr \ . \tag{1}$$

Here, $\psi \in \mathbb{R}^m$ is given vector; $\langle \cdot, \cdot \rangle$ is scalar product in $\mathbb{R}^m$; C is given value; $G : \mathbb{R}_+ \to \mathbb{R}$ is given function. The aim of control consists in minimization of guaranteed result of the quality index (1).

The control problem is considered within the theory of guaranteed result optimization [2]. With the help of a linear change of variables, the control problem comes down to a homogeneous differential game [3]. An optimal control existence theorem is proved. Necessary and sufficient conditions are found under which an admissible control is optimal.

References

1. Krasovskii, N.N.: The theory of motion control. Nauka, Moscow (1968). (in Russian)
2. Krasovskii, N.N., Subbotin, A.I.: Positional differential games. Nauka, Moscow (1974). (in Russian)
3. Ukhobotov, V.I.: One type differential games with convex goal. Trudy Instituta Matematiki I Mekhaniki UrO RAN. 16 (5), 196–204 (2010). (in Russian)

* The research was supported by Grant of the Foundation for perspective scientific researches of Chelyabinsk State University (2017)