Identification of Inactive Constraints in Convex Optimization Problems ⋆

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We consider the problem of convex optimization in the next form

$$\begin{array}{l} f_0(x) \to \min, \\ f_i(x) \le 0,\ x \in \mathbb{R}^n, \end{array} \tag{1}$$

where $f_i(x),\ i = 1, ..., m$ – convex not necessarily differentiable functions. We apply the simplex embeddings method [1]-[3] to solve the problem (1). This method is the analog of the well known ellipsoid method [4], [5]. In simplex embeddings method it is used $n-$dimensional simplexes instead of ellipsoids to localize the solution of a problem.

Consider the main idea of the method. Suppose that we have the start simplex S_0 on the step $k = 0$. This simplex contains the feasible set of the problem (1). We find the center $x^{c,0}$ of the simplex S_0 and construct the cutting hyperplane $L = \{x : g^T \left(x - x^{c,0}\right) = 0\}$ through the center, where $g \in \mathbb{R}^n$ is the subgradient of the objective function. Then we move to the next step $k = k + 1$ and immerse the simplex part that contains the solution to the problem into the new simplex S_1 which has the minimal volume. Using of this procedure let us construct simplexes that have less volumes than previous ones. Such algorithm provides the localization of the problem solution consistently. We stop the method when the simplex volume becomes quite small.

An exeption of inactive constraints is the unique feature of the simplex embeddings method. Suppose that we have a simplex which contains the solution of the problem (1). Denote by v^j the j-th simplex vertex, where $v \in \mathbb{R}^n$, $j = 1, ..., n+1$. Also we have the list of the constraints $f_i(x) \le 0, i = 1, ..., m$ from the problem (1). We must calculate the value of each function $f_i(x)$, $i = 1, ..., m$ in each simplex vertex to identify inactive constraints. Then we check further condition:

$$f_i\left(v^j\right) < 0, \quad i = 1, ..., m, \quad j = 1, ..., n + 1. \tag{2}$$

The i-th constraint from (2) is inactive if the inequality (2) is correct for each simplex vertex.

We apply the technique of inactive constraints identification on the set of convex programming problems. The results of numerical experiments are given in this paper.

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