Large-scale problems of discrete optimizaton: an asymptotically optimal approach vs a curse of dimensionality

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Keywords: capacitated FLP, exact algorithm, tree network, line graph

For discrete optimization problems, the main factor determining the feasibility of algorithms for their solution is the dimension (the length of the entry record) of the problem, which in the years 50-70 at the last century was associated with "a curse of dimensionality".

In contrast, within the framework of an asymptotically optimal approach to (approximate) solution of difficult problems of discrete optimization, the dimension of the problem is our friend and confederate.

We are talking about problems such as routing, multi-index assignments, clustering, allocation, extreme problems on graphs and networks, and so on.

Usually, these problems are difficult to solve (NP-hard), which causes the urgency of developing effective approximation algorithms with guaranteed estimates of the quality of their work — time complexity, performance ratio (or relative error), reliability of operation.

The curse of dimensionality is a problem associated with the exponential increase in the time of the solution of the problem due to the increase in the dimensionality of space (Richard Bellman, 1961).

Estimation of the relative error of the algorithm A of the solution on deterministic inputs of size n are called $\varepsilon_A(n)$ such that $|W_A(I) - OPT(I)|/OPT(I) \leq \varepsilon_A(n)$ is true on any input of I, where $OPT(I)$ and $W_A(I)$ are values of the objective functions, optimal and found by the algorithm A at the input I.

For problems on random inputs, the quality of the algorithm is also characterized by the probability of failure $\delta_A(n)$ which equal to the fraction of cases when the algorithm A does not guarantee the solution with the declared error. The better the algorithm, the smaller $\varepsilon_A(n)$ and $\delta_A(n)$.

The first example of an algorithm with an almost always guaranteed relative error is more than half a century ago! (Borovkov A. A. To the probabilistic formulation of two economic problems // DAN SSSR. 1962. 146 (5). 983–986): TSP and AP.

The algorithm A with the estimates ε_n and δ_n is asymptotically optimal if $\varepsilon_A(n) \to 0$ and $\delta_A(n) \to 0$ when $n \to \infty$.

⋆ The authors are supported by the Russian Foundation for Basic Research grants 16-07-00168 and 15-01-00976.

The report presents examples of the implementation of an asymptotically optimal approach to the solution of some large-dimensional problems of discrete optimization in the operations research in which the author has been directly involved in the past half century.