On optimization approach to solving nonlinear equation systems*

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Consider the following nonlinear equation system [1]:

$$f_i(x) = 0, \quad i = 1, 2, \ldots, m, \tag{1}$$

where $f_i \colon I\!R^n \to I\!R$, $i = \overline{1, m}$, are d.c. functions [2], which can be represented as a difference of two convex functions. Further, we reduce system (1) to nonsmooth optimization problem (see [1]), where objective function $F(\cdot)$ is also the d.c. function [2]. Furthermore, we consider the following d.c. representation: $F(x) = G(x) - H(x)$, where the function $G(\cdot)$ are nonsmooth and function $H(\cdot)$ are differentiable.

For solving the optimization problem we apply the Global Search Theory [3] based on necessary and sufficient global optimality conditions. Note that global search method includes two principal parts: local search and procedures of improving a critical point $z \in I\!R^n$ (i.e procedures for funding a point $u \in I\!R^n$ such that $F(u) < \zeta$, where $\zeta := F(z)$) provided by a local search method.

To this end for a fixed vector $y \in I\!R^n$ it is necessary to solve the following nonsmooth convex auxiliary (partially linearized) problem (both on every step of the special local search method and on the stage of improving a critical point). In order to perform it, we solve the nonsmooth auxiliary problems via the smooth convex problems, increasing the dimension from n up to $(m + n)$.

The computational experiments were carried out on test problems with dimension up to 100. For solving auxiliary problems we apply existing methods and software (for instance, IBM ILOG CPLEX). In addition, we compare the effectiveness of developed algorithms with rather popular solvers.

References

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