Multicommodity flows model for the Pacific Russia interregional trade

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The paper describes a model of trade flows among the territories of Pacific Russia based on multi-commodity network flow equilibrium approach based on Boyce and others [1] equilibrium modeling approach to simulate interregional multi-product flows within the transport system.

Let z_{ij}^{rm} is an unknown volume of trade of the product of r-th type delivered from the i-th region to the j-th by m-th type of transport called "mode". The transportation network given among regions which is desribed by nodes and arcs of a network. Let $c_l^{(m)}(y)$ is the cost of flow y moving on arc l by m-th mode, and $g(z,t)$ is transportation costs of moving volume z of a product between regions i and j which also depends of the distance t between them.

Then we introduce h_p^m as an unknown total flow along the path p for mode m, L_m is a number of network arcs for mode m, $f_l^{rm} = \sum\limits_{p=1}^{P_m} d_{lp}^m h_p^{rm}$ is a flow along the arc l for mode m where P_m is a number of all possible paths between any pair of regions for mode m. Numbers d_{lp}^m equals to one if for fixed m the arc l is a part of path p otherwise they equal zero.

Equilibrium distribution of flows over the network can be expressed as complementary slackness conditions $h_p^m(\sum\limits_{l}^{L_m} c_l^m(f_l^m)d_{lp}^m - u_{ij}^m) = 0$, and $z_{ij}^{rm}(g(z_{ij}^{rm}) + u_{ij}^{rm}) = 0$ which reflects the principle of Wardrop for network equilibrium that, firstly, if the flow h_p^m along the path p is not equal to zero i.e. $h_p^m > 0$, then the total cost of flow moving $c_p^m = \sum\limits_{l=1}^{L_m} c_l^m(f_l^m)d_{lp}^m$ on all the paths p are equal to the equilibrium value costs u_{ij}^{rm} which are independent of the path. Secondly, if for some way between the regions i and j total expenses is strictly greater than equilibrium value costs, i.e. $c_p^m > u_{ij}^m$, then all $h_p^m = 0$. All these mean that none of the unloaded paths do not have a lower cost for transportation than c_p^m.

References

1. Ham H., Kim T.J., Boyce D.: Implementation and estimation of a combined model of interregional, multimodal commodity shipments and transportation network flows. Transport. Res. B. 39, 65–79 (2005)