An Approach to Solving Control Problems of Heat Processes with Phase Transitions

Alla Albu[1] and Vladimir Zubov[1,2]

[1] Dorodnicyn Computing Centre, FRC CSC RAS
[2] Moscow Institute of Physics and Technology alla.albu@yandex.ru

The class of problems in which a material under analysis transforms from one phase into another with heat release or absorption is of great theoretical and practical interest. Such problems arise in studies of many phenomena, among which melting and solidification are the most important and widespread. The problems arising in practice do not reduce to the description of processes involving phase transitions, but also include optimal control of these processes. Optimal control of processes involving phase transitions is interpreted as the choice of some process parameters (controls) in such a way that the process is as close as possible to a given scenario; for example, the behavior of the liquid-solid phase boundary or a function of temperature in some domain is closest to a required behavior. An effective approach to solving this type of problems was developed and applied in practice by the authors of this article. The efficiency of the method is explained by the simultaneous use of three basic elements.

First, during the solution of the initial boundary value problem that describes the process of heat transfer, the statement of a boundary value problem in terms of temperature is reformulated in terms of enthalpy. The reason for this is the fact that, as one intersects the phase boundary, the temperature changes smoothly while the enthalpy undergoes a jump change.

The second element of this approach is a special iterative algorithm proposed by the authors for solving nonlinear systems of finite-difference equations obtained as a result of approximating the initial-boundary value problem. The new iterative algorithm is much more efficient than algorithms used earlier: the modified Jacobi method and the modified Gauss-Seidel method.

Optimal control problems for thermal processes with phase transitions are usually solved numerically using gradient methods. To ensure the efficiency of a gradient method, the gradient of the cost function has to be computed to high accuracy. The third element of the proposed approach is connected with the fact that the gradient of the cost function of the optimal control problem is calculated using the Fast Automatic Differentiation technique. This method offers canonical formulas that produce the exact value of the gradient in a discrete optimal control problem.

The above-indicated approach is illustrated based on the example of the solution of the problem of controlling the phase boundary evolution in the substance solidification process in foundry practice. A mathematical model of this process is underlain by a three-dimensional unsteady two-phase initial-boundary value problem of the Stefan type.