

# Approximation schemes for the Euclidean CVRP with non-uniform demand and time windows

Michael Khachay<sup>1\*</sup> and Roman Dubinin<sup>2</sup>

<sup>1</sup> Krasovskii Institute of Mathematics and Mechanics,  
Ural Federal University, Ekaterinburg, Russia  
Omsk State Technical University, Omsk, Russia  
`mkhachay@imm.uran.ru`

<sup>2</sup> Ural Federal University, Ekaterinburg, Russia  
`romandubinin94@gmail.com`

Capacitated Vehicle Routing Problem (CVRP) is the well-known combinatorial optimization problem introduced by G. Datzig and J. Ramser in their seminal paper [1]. As known (see, e.g. [2]), this problem is NP-hard even in finite dimensional Euclidean spaces. Although the problem is hardly approximable in general, its geometric settings can be approximated rather well. Most of the known results in this field date back to the famous papers by M. Haimovich and A. Rinnoy Kan [3] and S. Arora [4]. Above these results, the most recent seem to be the quasipolynomial time approximation scheme proposed in [5] for the Euclidean plane and extended in [6] to the case of finite number of non-intersecting time windows, and (to the best of our knowledge) the first polynomial time approximation scheme introduced in [7] for the CVRP formulated in  $d$ -dimensional Euclidean space for an arbitrary  $d > 1$ .

In this paper, we propose an extension of the results of [6] to the case of  $d$ -dimensional Euclidean spaces and non-uniform customer demand following the general framework developed in [7].

## References

1. Dantzig, G.B., Ramser, J.H.: The truck dispatching problem. *Management science* 6(1), 80–91 (1959)
2. Toth, P., Vigo D. (eds.): *Vehicle Routing: Problems, Methods, and Applications*, Second Edition. SIAM, Philadelphia, PA, USA (2014)
3. Haimovich, M., Rinnooy Kan, A.H.G.: Bounds and heuristics for capacitated routing problems. *Mathematics of Operations Research* 10(4), 527–542 (1985)
4. Arora, S.: Polynomial time approximation schemes for euclidean traveling salesman and other geometric problems. *Journal of the ACM* 45(5), 753–782 (1998)
5. Das, A., Mathieu, C.: A quasipolynomial time approximation scheme for Euclidean capacitated vehicle routing. *Algorithmica* 73, 115–142 (2015)
6. Song, L. et al. Approximation schemes for Euclidean vehicle routing problems with time windows. *Journal of Combinatorial Optimization* 32(4), 1217–1231 (2016)
7. Khachay, M., Dubinin, R. PTAS for the Euclidean Capacitated Vehicle Routing Problem in  $R^d$ . LNCS 9869, 193–205 (2016)

---

\* Supported by RSCF grant 14-11-00109