

A parameterized approximation algorithm for the mixed capacitated arc routing problem: Theory and experiments

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The mixed capacitated arc routing problem (MCARP) models the task of finding minimum-cost tours for servicing links in transportation networks using a fleet of vehicles with equal capacity. Herein, *mixed* means that the transportation network may have undirected edges and directed arcs.

Problem (MCARP).

Instance: A mixed graph $G = (V, E, A)$ with edge set E and arc set A , a depot vertex $v_0 \in V$, traversal costs $c: E \cup A \rightarrow \mathbb{N} \cup \{0\}$, demands $d: E \cup A \rightarrow \mathbb{N} \cup \{0\}$, and a vehicle capacity Q .

Goal: Find a set W of closed walks in G , each passing through the depot vertex v_0 , and a *serving function* $s: W \rightarrow 2^{E \cup A}$ determining for each walk $w \in W$ the subset $s(w)$ of the edges and arcs it *serves*, such that

- $\sum_{w \in W} c(w)$ is minimized, where $c(w) := \sum_{i=1}^{\ell} c(e_i)$ for $w = (e_1, e_2, \dots, e_{\ell})$,
- $\sum_{e \in s(w)} d(e) \leq Q$, and
- each edge or arc e with $d(e) > 0$ is served by exactly one walk in W .

If $A = \emptyset$, then MCARP is $(7/2 - 3/W)$ -approximable. Otherwise, even if $E = \emptyset$, MCARP is as least as hard to approximate as the n -vertex metric asymmetric Traveling Salesperson problem (Δ -ATSP), for which the best known is a polynomial-time $O(\log n / \log \log n)$ -approximation. Conversely, we prove:

Theorem (van Bevern, Komusiewicz, Sorge [1]). Any polynomial-time $\alpha(n)$ -approximation algorithm for the n -vertex Δ -ATSP yields a polynomial-time $O(\alpha(C))$ -approximation algorithm for MCARP, where C is the number of weakly connected components in the graph induced by positive-demand arcs and edges.

The number C is small in many applications and benchmark data sets (where, usually, $C = 1$). We present some heuristic enhancements that help our algorithm to outperform many previous polynomial-time heuristics.

Literature

- [1] René van Bevern, Christian Komusiewicz, and Manuel Sorge. A parameterized approximation algorithm for the mixed and windy capacitated arc routing problem: theory and experiments. *Networks*, 2017. In press. [doi:10.1002/net.21742](https://doi.org/10.1002/net.21742).

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