Application of the linearization method for solving quantile optimization problems

Sofia Vasileva and Yuri Kan

Moscow Aviation Institute, 125993 Moscow, Russia
{sofia_mai, yu_kan}@mail.ru

Abstract. The quantile optimization problem is studied in the case where random parameters are small. The linearization method is presented which allows us to replace the original nonlinear loss function in the quantile statement by its model linear in random parameters.

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The problem under consideration is minimization of the quantile criterion in the general nonlinear formulation. The quantile criterion is defined as a quantile of the given confidence level for the distribution of a nonlinear loss function which depends nonlinearly on the deterministic decision vector and the vector of random parameters.

The vector of random parameters is assumed to be small. This fact is modeled as an element-by-element product of the vector of small deterministic parameters by a vector of random parameters with a given joint distribution.

The following linearization method is discussed in the report. The original loss function is replaced by its linearized model. The linearization is performed by expanding the original function in a Taylor series w.r.t. the random parameter vector in a zero neighborhood. Thus, the original problem reduces to the problem of quantile optimization of a loss function linear in random parameters. The error that arises in such a replacement has the order of the square of the vector norm of small deterministic parameters.

The linearized quantile optimization problem can be reduced under some regularity conditions to an equivalent minimax problem where the maximum is taken w.r.t. realization of random vector over the kernel of the probability measure. The kernel is intersection of all the closed α-confidence half-spaces. For the Cauchy distribution, normal and uniform distributions, the kernel can be constructed analytically. In cases where such a construction can be hardly found we use an external approximation of the kernel by a polyhedron with a given number of vertices. Two algorithms are presented for this aim.

Let us consider the practically important case where the original loss function depends linearly on the decision vector and the feasible decision set is a polyhedron. Such a model is typical for Portfolio Selection. In this case the abovementioned minimax problem where the kernel is replaced by its approximating polyhedron can be reduced to the equivalent LP problem of large dimension. The corresponding example for selection of Russian bonds portfolio is discussed.

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