## Minimizing machine assignment costs over $\Delta$ -approximate solutions of $P||C_{\max}$

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Bi-criteria lexicographical minimization problems with the makespan as the primary objective and the total machine assignment costs as the secondary objective have been recently introduced to the scheduling research, and polynomial time  $(\Delta, 1)$ -approximation algorithms have been suggested for their solution [1]. We study a problem of minimizing the total machine assignment cost over the  $\Delta$ -approximate solutions of the makespan minimization problem. We prove that this new problem is strongly NP-hard and pseudo-polynomially non-approximable in general. A polynomial time approximation algorithm with a guaranteed approximation ratio is presented for a special case where the ratio between the maximal and minimal costs associated with the machines is bounded. An  $O(mn^{2k})$  time dynamic programming algorithm is presented for another special case in which the number k of distinct job processing times is fixed.

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## References

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