On the optimal non-destructive system exploitation problem in certain general setting

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The report is devoted to the use of a renewable resource based on its withdrawal from the certain system, not leading to its destruction (for example, the harvest in ecology). This problem is very relevant at present [1]. Our aim is to present a unified approach to managing such systems.

Let the mapping F is concave on the nonnnegative cone $\mathbb{R}^n_+$ of the space $\mathbb{R}^n$. The purpose of optimization is to obtain the maximum admissible total effect

$$\tilde{c} = \max\{\langle c, x\rangle \mid x \in \overline{U}\}, \tag{1}$$

where $c, x \in \mathbb{R}^n_+$, $x = [x_1, x_2, \ldots, x_n]$, $\langle\cdot,\cdot\rangle$ — the scalar product, $\overline{U}$ — the closure of the set $U = \{u \in \mathbb{R}^n_+ \mid X_0(u) \neq \varnothing\}$,

$$X_0(u) = \{x_0 \in \mathbb{R}^n_+ \mid \inf_{t=0,1,2\ldots} x^i_t(x_0, u) > 0 \ (\forall\, i = 1, \ldots, n)\},$$

$$x_{t+1}(x_0, u) = \big(F\big(x_t(x_0, u)\big) - u\big)^+, \quad t = 0, 1, 2\ldots \tag{2}$$

Here $x_t(x_0,u) = [x^t_1(x_0,u), x^t_2(x_0,u), \ldots, x^t_n(x_0,u)]$, $x_0(x,u) = x$, $x^+_i = \max\{x_i,0\}$, $x^+ = [x^+_1, x^+_2, \ldots, x^+_n]$.

Denote by $\tilde{u}$ and $\widetilde{U}$ the optimal vector and optimal set of problem (1), respectively; and by N_u and $\hat{N}_u$ the sets of nonzero fixed points and positive fixed points of the mapping $F_u(x) = \big(F\big(x_t(x_0, u)\big) - u\big)^+$, respectively. The behavior of the iterative process (2) is described by the following way:

a) if $N_u = \varnothing$ then $\lim_{t\to+\infty} x_t = 0$ regardless $x_0 \geq y(F)$ where $y(F)$ is the maximum fixed points of the mapping F;

b) if $\hat{N}_u \neq \varnothing$, then $\lim_{t\to+\infty} x_t = y(u)$, where $y(u)$ is the maximum fixed points of the mapping F_u;

c) $X_0(u) \neq \varnothing$ if and only if $\hat{N}_u \neq \varnothing$.

Thus, the equality $\overline{U} = \{u \in \mathbb{R}^n_+ \mid N_u \neq \varnothing\}$ is valid and the problem (1) can be reduced to the following convex program:

$$\max\{\langle c, u\rangle \mid x = F(x) - u,\ x \geq 0,\ u \geq 0\}.$$

Investigated optimal solutions of this problem for some nonlinear generalizations used in mathematical ecology models.

References

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2. Smirnov, A. I. Equilibrium and stability subhomogeneous monotone discrete dynamical systems.Ekaterinburg: UIECL, 2016. 320 pp (in Russian).