On the regularization for improper problems of convex programming

Vladimir D. Skarin

Krasovskii Institute of Mathematics and Mechanics UB RAS, Ekaterinburg, Russia,
Ural Federal University, Ekaterinburg, Russia
skavd@imm.uran.ru

Consider the problem of convex programming (CP)

$$\min\{f_0(x) \mid x \in X\}, \tag{1}$$

where $X = \{x \mid f(x) \leq 0\}$, $f(x) = [f_1(x), \dots, f_m(x)]$, the functions $f_i(x)$ are convex in $\mathbb{R}^n$ for $i = 0, 1, \dots, m$. We consider problem (1) with inconsistent constraints: $X = \varnothing$. Such models form [1] a very important class of improper problems (IP) of mathematical programming. Let $X_\xi = \{x \mid f(x) \leq \xi\}$, $E = \{\xi \in \mathbb{R}^m_+ \mid X_\xi \neq \varnothing\}$ and $\bar{\xi}_p = \arg\min\{\|\xi\|_p \mid \xi \in E\}$, $\|\cdot\|_p$ denotes p-norm in $\mathbb{R}^m$.

Along with (1), we consider the problem

$$\min\{f_0(x) \mid x \in X_{\bar{\xi}_p}\}. \tag{2}$$

If $X \neq \varnothing$ in problem (1) then we have $\bar{\xi}_p = 0$ and problems (1) and (2) coincide. Otherwise, (2) is an example of possible correction for IP (1), and we may accept the solution of (2) as generalized (approximative) solution of IP (1).

To problem (2) we assign the auxiliary problem

$$\min_x\{F_\alpha(x, r) = f_0(x) + \alpha\|x\|_2^2 + r\|f^+(x)\|_p,\ r > 0\}. \tag{3}$$

The function $F_\alpha(x, r)$ is strongly convex with respect to $x \in \mathbb{R}^n$. Hence, problem (3) has a unique solution for every $r > 0$ and $\alpha > 0$, including the case $X = \varnothing$, in contrast with problem (2). Therefore, the function $F_\alpha(x, r)$ can be used for the analysis of IPCP.

In this report we obtain the estimates characterising the convergence of $F_\alpha(x, r)$ minimizer to approximate solution of the IP. Here, we consider special the situations as follows

a) in problems (1) and (2) instead of the functions $f_i(x)$ some approximations $f_i^\varepsilon(x)$ are known;

b) in problems (2) the different norms for $p = 1, 2, \infty$ are chosen;

c) in problems (2) the vector $\bar{\xi}_p$ is undefinable;

d) the problems (2) is unsolvable.

References

1. Eremin, I. I., Mazurov, Vl. D., Astaf'ev, N. N.: Improper Problems of Linear and Convex Programming. Nauka, Moscow. In Russian (1983).