

NEW GUARANTEED ERRORS GREEDY ALGORITHMS IN CONVEX DISCRETE OPTIMIZATION PROBLEMS IN TERMS OF STEEPNESS UTILITY FUNCTIONS

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Abstract. As is known (see, [1, 2]), a guaranteed error estimate for the gradient algorithm as applied to some discrete optimization problems can be expressed in terms of the steepness of the utility functions. We obtain improved guaranteed estimates for accuracy of the gradient algorithm.

We consider the following convex discrete optimization problem A : find

$$\max\{f(x) : x = (x_1, \dots, x_n) \in P \subseteq Z_+^n\},$$

where $f(x) \in \mathfrak{R}_\rho(Z_+^n)[1]$, $f(x)$ is a nondecreasing function on the set P , P is an ordinal-convex set [3]. Let $x^g = (x_1^g, \dots, x_n^g)$ be the gradient solution (the gradient maximum of the function $f(x)$ -on the set P) of the problem A [1, 3].

Let c be steepness of the function $f(x)$ on the set P [1,2].

Theorem. Let $f(x) \in \mathfrak{R}_\rho(Z_+^n)$ be a nondecreasing on the set P function. Then the global maximum x^* and the gradient maximum x^g of the function $f(x)$ on the set $P \subseteq Z_+^n$ satisfy the relation

$$\frac{f(x^*) - f(x^g)}{f(x^*) - f(0)} \leq \left(1 - \frac{1}{1 + (1 - c)(h - 1)} \right)^r - B,$$

where $h = \max\{h(x) = \sum_{i=1}^n x_i : x = (x_1, \dots, x_n) \in P\}$, $0 \leq B \leq 1$, $r = \min\{h(x) - 1 : x \in Z_+^n \setminus P\}$.

References

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3. M.M. Kovalev: Matroids in discrete optimization (Russian), Minsk, 1987, 222 p.